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Dynamic interaction models of economic equilibrium

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ABSTRACT

The paper develops a stochastic dynamic model of economic equilibrium with locally interacting agents. The main focus of the study is on the modeling of market interactions – those arising in connection with commodity exchange and regulated by price mechanisms. The mathematical framework is a control theory for random vector fields on directed graphs. The graphs involved serve to describe the spatio-temporal structure of commodity flows in the system. The main results are concerned with the existence, uniqueness and stability of stochastic dynamic equilibria.

1. Introduction

The general objective of this study is to develop stochastic models of dynamic economic equilibrium taking into account local interactions between economic agents. The classical theory allows individuals to interact only on the macrolevel – through a price system prevailing in the market. However, the real economic process involves by and large direct contacts between its participants, and prices generally depend on when and where the trading takes place. It is therefore of interest to provide equilibrium models taking into account possibilities of, as well as restrictions on, direct interactions between agents. The restrictions (specifying the notion of 'locality') may be of various types: certain individuals may only be able to exchange commodities with certain others; some may only communicate with a group of the others; there may be spatial, temporal and informational constrains.

In the last three decades, a large amount of work has been done on developing various kinds of local interaction models. Different views on the problem and different formal settings have been proposed. A starting point for this strand of literature was a pioneering paper by Föllmer (1974) (see also Karmann, 1976). Contributions to the field were made by Allen (1982), Aoki (1996), Blume (1993), Blume and Durlauf (2003), Brock and Durlauf (2001a, b), Brock and Hommes (1997), Durlauf (1993, 1997), Glaeser and Scheinkman (2003), Hommes (2001, 2006), Horst and Scheinkman (2006), Ioannides (1997, 2006), Kirman (1997a, b), Kirman et al. (1986), Kirman and Vriend (2001), Lux (1998), Lux and Marchesi (1999), Verbrugge (2003), Weisbuch et al. (2000), and others.

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In spite of the variety of approaches and frameworks, two common features are characteristic for this large body of literature. (a) The main achievements pertain basically to models which aim at the analysis of *non-market* (*social*) *interactions*. (b) The mathematical apparatus used and the results obtained are often suggested by the methods developed in statistical physics for the analysis of multicomponent random systems (e.g., Preston, 1974). In particular, an important role is played by the techniques of Gibbs–Markov fields on undirected graphs, such as n-dimensional lattices. Both features are clearly visible in the seminal paper by Föllmer (1974). In Föllmer's model, agents live at nodes of a lattice, locally interact in terms of their exogenous random characteristics, but the notion of a market equilibrium is in essence global. This notion is defined in terms of a balance between aggregate supply and demand, with one price system for all the agents. The results in Föllmer (1974), as well as in a number of other papers in the above literature, involve an economic interpretation of the celebrated results on phase transitions for Gibbs fields (e.g., Sinai, 1982) – a source of inspiration for generations of researchers.

Although our direction of studies is also aimed at the modeling of local interactions in economics, its primary focus is different. We concentrate on the modeling of *market interactions* – those which arise in connection with commodity exchange and are regulated by price mechanisms. The results we obtain extend to this framework results of general equilibrium theory and the theory of economic growth. Technically, our approach does not rely upon the apparatus of statistical physics, although some parallels (pertaining to random fields) can be traced. The models we develop are based on stochastic control schemes involving random fields on directed structures, such as directed graphs, partially ordered sets, etc. These control schemes generalize to the local interaction case those developed in connection with the conventional stochastic models of economic dynamics and equilibrium (see Arkin and Evstigneev, 1987).

In the present paper we examine a stochastic equilibrium model in which markets – where agents interact in the process of commodity exchange – are separated in space and time. The model is specified in terms of a fixed directed graph, T. The vertices of T correspond to *economic units* (e.g., regions) or *agents* representing them. The agents act at certain moments of time. They produce and consume commodities and deliver them to other agents. The directed arcs of the graph T describe the spatio-temporal structure of the directions of commodity flows in the system. Different agents in the economy are influenced by different random factors and possess different information. It is supposed that the stochastic structure of the model is in a sense compatible with the structure of the given graph T (if t depends on s via commodity supplies, random factors affecting s may also affect t).

We analyze *equilibrium states* of the economy, i.e., those states in which all the agents implement their most preferred production and consumption decisions (given the local equilibrium prices) and balance constraints for the commodity flows are satisfied. The main results are existence and uniqueness theorems for such states. The results generalize those obtained in our previous work (Evstigneev, 1991; Evstigneev and Taksar, 1994, 1995), dealing with the case of a finite graph, to infinite graphs. This generalization requires some hypotheses regarding the infinite graph T. In particular, we need certain restrictions on the 'branching rate' of T and the assumption that T is well approximable in a proper sense by its finite subgraphs. These assumptions are used when passing to the limit as the number of nodes of T tends to infinity. A key role at this stage of the analysis is played by the stability results – *turnpike theorems* – for equilibria established in Dempster et al. (1998).

The study of infinite graph models is motivated primarily by the fact that they reflect the idea of a 'large' economy, developing over a long time interval. They provide a framework for analyzing stability and other asymptotic properties of growth in the dynamic equilibrium context. The present paper is a step in our program of extending to the graph models the key results of the mathematical theory of optimal and equilibrium growth over an infinite time horizon, as developed by Gale (1956, 1967, 1968), Nikaido (1968), McKenzie (1986, 1998), Brock (1970), Brock and Mirman (1973), Brock and Haurie (1976), Radner (1971, 1973, 1982), Polterovich (1978), Bewley (1981, 1982), Majumdar and Zilcha (1987), Dana and Le Van (2006), and others. This theory – rich in content and mathematically elegant – belongs to the classics of mathematical economics. The focus on this range of questions is one of the main distinctions of the present line of studies from other research dealing with network models in economics, regional science, games, and operations management (e.g., Samuelson, 1952; Nagurney, 1993; Shapiro and Varian, 1998; Batten and Boyce, 1986; Nijkamp and Reggiani, 1998; Bernstein et al., 1993).

We consider in this work optimal and equilibrium control problems involving random fields. By a 'random field' one usually means a random function whose domain does not have a natural structure of linear ordering (a Euclidean space, a manifold, a graph, etc.). The present study deals with functions of this kind defined on directed graphs or partially ordered sets. The control scheme which lies in the basis of our model was proposed in Evstigneev (1988, 1991) with the primary view to economic applications. It generalizes, in particular, the framework of *stochastic convex dynamic programming* (Dynkin, 1972; Rockafellar and Wets, 1975) and includes as special cases Gale's (1956, 1967) multisector models of economic growth and their stochastic analogues proposed by Dynkin (1971, 1972, 1976), Radner (1971, 1973, 1982) and others – see surveys in Arkin and Evstigneev (1987) and Evstigneev and Schenk-Hoppé (2006). (It does not include as special cases those models which are based on the theory of Gibbs fields – see the references above.) For related classes of optimization problems involving random functions on partially ordered sets see Krengel and Sucheston (1981) and Mandelbaum and Vanderbei (1981).

It should be noted that the general theory of random fields on directed structures is much less developed than its counterpart pertaining to undirected structures. In particular, the Markov property of random functions, that can be completely characterized by using powerful Gibbs methods in the case of undirected graphs, is much less explored in