Journal of Mathematical Economics 46 (2010) 1211–1221

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

Demographic-economic equilibria when the age at motherhood is endogenous☆

Hippolyte d'Albis[a], Emmanuelle Augeraud-Véron[b], Katheline Schubert[c,*]

[a] Toulouse School of Economics (LERNA), France
[b] University of La Rochelle (MIA), France
[c] Paris School of Economics and University Paris 1 (CES), France

ARTICLE INFO

Article history:
Received 22 July 2008
Received in revised form 1 September 2010
Accepted 9 September 2010
Available online 17 September 2010

Keywords:
Demographic economics
Fertility choices
Monetary equilibrium

ABSTRACT

In this article, we study the joint dynamics of the demography and the economy. We explore how economic conditions affect fertility choices, and in return how the population growth rate affects both financial and labor markets. Our main contribution is to consider a realistic demographic setup that allows characterizing the age at which individuals decide to give birth to their children. In such a framework, we aim at studying the existence of an equilibrium. We notably prove there exists a monetary steady state if the average age of consumers is greater than the average age of producers.

1. Introduction

In demographics, there is a long tradition of modeling population dynamics, dating back to the pioneer works of Lotka (1907, 1922). Stable population theory studies the dynamics of an age distribution once fertility and mortality patterns are held constant. Generalizations have then been proposed to take into account deterministic or stochastic changes in fertility and mortality. We propose to extend the traditional stable population model to endogenous fertility behaviors hinging on a trade-off between the utility derived from having children and the costs they induce. A key variable affecting both utility and costs is the age at which women become mothers. As pointed out by Gustafsson (2001), it is notably the age at first birth that is the main explanatory variable for the rapid decrease in fertility in developed countries. Using the data from the Human Fertility Database (2010), it is striking to see the negative correlation between the cohort's mean first maternal age and their total fertility rate. For instance, in the US, the mean age was slightly greater than 24 for cohorts born in 1918 as well as for those born in 1956, while it reached 22 and less for the cohorts, born between 1934 and 1940, that participated to the post-war baby-boom. In Fig. 1 are represented the dynamics of the mean age at first birth and of the total fertility rates for cohorts born between 1918 and 1966.

We consider an overlapping generations model with continuous trading in which individuals work, consume and decide the age at motherhood. This choice affects labor participation and aggregate economic variables. We study the monetary equilibrium such that aggregate assets are positive. The main departure from Samuelson (1958) and the subsequent literature is that the population growth rate is endogenous.

☆ We thank Noel Bonneuil, David de le Croix, the participants to the VID Viennese Vintage Workshop and the Overlapping Generations Days at Vielsalm, and two anonymous referees for their comments. The usual disclaimer applies.

* Corresponding author.
E-mail address: schubert@univ-paris1.fr (K. Schubert).

0304-4068/$ – see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.jmateco.2010.09.001

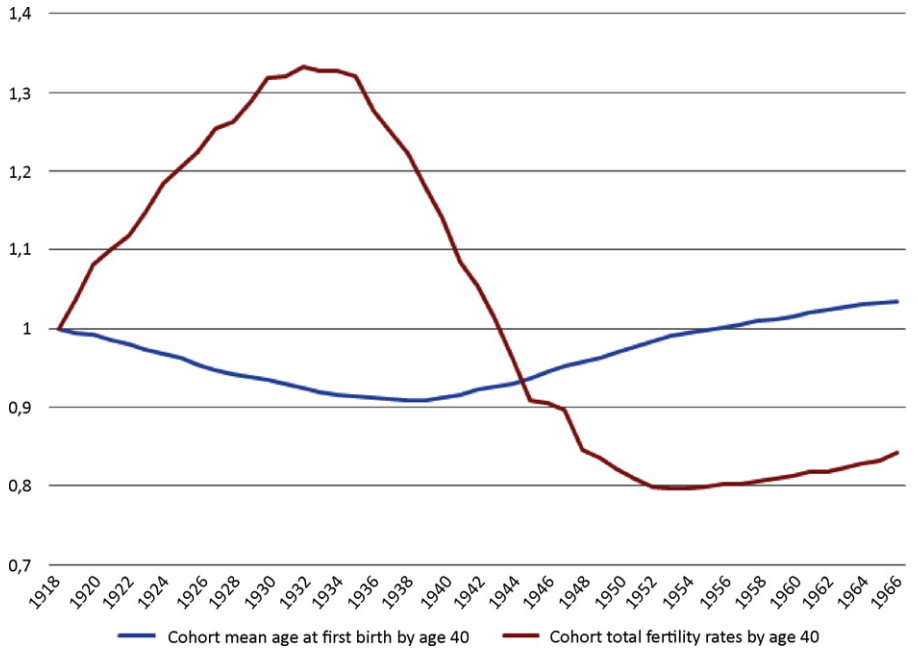

Fig. 1. Evolution of cohorts's mean age at birth and fertility, USA. Source of data: Human Fertility Database (2010).

The intertemporal equilibrium is shown to be the solution of a non-linear functional differential equation of mixed-type. The dynamics is indeed affected by discrete delays and advances. As in Boucekkine et al. (2002) and d'Albis and Augeraud-Véron (2007), delays are generated by the vintage structure of the population while advances rely on the expectations of the individuals. Moreover, with an endogenous age at motherhood, some of the delays and advances are state-dependent.

We characterize the steady states of our economy and focus on the monetary steady state. It is indeed well-known in the Samuelson (1958) setting that monetary steady states always appear as candidates for the equilibrium. In our framework with endogenous fertility, we show that it is not necessarily the case and that it depends on the marginal impact of the age at motherhood on human wealth. Moreover, we show that the condition initially exhibited by Arthur and McNicoll (1978) in a framework with exogenous population growth rate, still holds and is even necessary and sufficient for the existence of the monetary steady state. This condition says that the difference between the average age of consumers weighted by their consumptions and the average age of producers weighted by their earnings should be strictly positive. We finally show that the population growth rate obtained at the monetary equilibrium is always lower than the one that would be chosen by a social planner, upon existence of this latter solution.

Section 2 presents the model describing the individual life-cycle behavior and the aggregation of both the population and the economy. Section 3 studies the monetary equilibrium. Section 4 concludes.

2. The model

2.1. Individual behavior

Individuals' lifetime is deterministic,[1] and of length 1. Let s be the birth date of the representative individual of generation s and $s+\tau(s)\in[s, s+1]$ the date at which the individual born at s gives birth to her $n(s)$ children. The number of children is supposed to be a decreasing function of the age at which the individual gives birth to her children. This assumption is captured by function f, which satisfies:

$$n(s)=f(\tau(s)),\quad f(\cdot)>0,\ f'(\cdot)<0. \tag{1}$$

[1] We are aware that introducing some uncertain lifetime in accordance with Yaari (1965) would be not only more realistic (Boucekkine et al., 2004; d'Albis, 2007) but also might influence the results as argued by recent studies linking fertility to health conditions (Kalemli-Ozcan, 2003; Boucekkine et al., 2009). Specific difficulties arising with uncertain lifetime make nevertheless this issue beyond the scope of the paper.