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Kalman's Expanding Influence in the Econometrics Discipline

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Abstract: Kalman's original work was quickly adopted and explored throughout the engineering disciplines after the publishing of his seminal paper in 1960 (Kalman 1960). However, it would be several decades before the utility of his framework was understood and adopted in the econometrics discipline, which had traditionally based its analysis on the Box-Jenkins ARIMA time series models. This paper surveys how Kalman's work has been applied in developing econometric models and how econometricians have extended and built upon Kalman's framework to develop new techniques to model complex economic and financial systems over the past 20-25 years.

Keywords: Kalman filters, economics

1. INTRODUCTION

Starting in the 1980s, econometricians began to realize that Kalman's regressions and the state-space models upon which they are based provide many advantages as a framework for their analyses. At the Fifth World Congress of the Econometric Society in August 1985, Andrew Harvey observed that “From the point of view of econometric modeling, the Kalman filter is of very little interest. It is simply a statistical algorithm that enables certain computations to be carried out for a model cast in state space form. The crucial point for the econometrician to understand is that the state space form opens up the possibility of formulating models that are much wider and richer than those normally considered. Furthermore it often allows the setting up of models that have a more natural interpretation and provide more useful information on the nature of the underlying economic processes.” (Harvey 1987)

Section 2 of this paper surveys the evolution of time series models used in the econometrics field and contrasts engineering and econometric applications of state-space models. Section 3 discusses selected applications of Kalman’s work that have been used by econometricians to advance the state of their modeling techniques.

2. ECONOMETRIC TIME SERIES MODELS

For purposes of this discussion, we will use the state space notation of Durbin and Koopman (Durbin & Koopman 2012):

$$\boldsymbol{y}_t = \boldsymbol{Z}_t\boldsymbol{\alpha}_t + \boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t \sim N(0, \boldsymbol{H}_t) \tag{1}$$

$$\boldsymbol{\alpha}_{t+1} = \boldsymbol{T}_t\boldsymbol{\alpha}_t + \boldsymbol{R}_t\boldsymbol{\eta}_t, \qquad \boldsymbol{\eta}_t \sim N(0, \boldsymbol{Q}_t) \tag{2}$$

where $\boldsymbol{y}_t$ is the observation, $\boldsymbol{\alpha}_t$ is the unobserved state vector, equation (1) is the *observation equation* and equation (2) is the *state equation*.

In both engineering and econometric applications, the observation equation is often extended to incorporate explanatory and intervention models:

$$\boldsymbol{y}_t = \boldsymbol{Z}_t\boldsymbol{\alpha}_t + \boldsymbol{X}_t\boldsymbol{\beta} + \boldsymbol{\varepsilon}_t, \qquad \varepsilon_t \sim N(0, \boldsymbol{H}_t) \tag{3}$$

where $\boldsymbol{\beta}$ is a an unknown vector of regression coefficients. In engineering applications, $\boldsymbol{X}_t$ is generally a controllable input while in econometric applications it generally represents explanatory variables. In engineering applications, the state equation often also contains an additional term involving the control inputs.

2.1 *Contrasting engineering and econometric models*

The first fundamental difference between engineering and econometric applications of the state space models is that in engineering models, the coefficient matrices ($\boldsymbol{Z}$, $\boldsymbol{T}$, $\boldsymbol{R}$, $\boldsymbol{H}$, and $\boldsymbol{Q}$) are generally known based on existing quantitative physical laws. Engineering applications then revolve around estimating the state from noisy observations where the state vector corresponds to a quantitative specification of the physical condition of the system being modelled.

In contrast, econometricians generally lack rigorous quantitative models of the systems that they wish to model. Instead, the goal is to define and estimate models that capture the behaviour of the systems being studied. As put by Andrew Harvey, “Econometrics is concerned with the estimation of relationships suggested by economic theory” (Harvey 1990). Econometricians typically refer to state space models as unobserved component models to indicate that the focus is on estimating attributes of the micro- or macro-economic systems that are postulated by their models but are not observable.

Because of these fundamental differences in the natures of the two types of models, econometric state-space models have focused on techniques for the estimation of the state space

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model coefficient matrices. The key advancement in making state space modeling valuable to econometricians was the 1965 development of a technique to use the Kalman filter to evaluate likelihood functions when a model is formulated in state space form (Schweppe 1965) which is often referred to as prediction error decomposition (Harvey 1989) and is based on defining the loglikelihood function based on the prediction error $\boldsymbol{v}_t = \boldsymbol{y}_t - \boldsymbol{Z}_t\widehat{\boldsymbol{\alpha}}_{t|t-1}$ instead of the observations $\boldsymbol{Y}_t = [\boldsymbol{y}_1' \dots \boldsymbol{y}_t']'$

$$\log L(\boldsymbol{Y}_n) = \sum_{t=1}^{n} \log p(\boldsymbol{y}_t|\boldsymbol{Y}_{t-1}) \qquad (4)$$

$$= -\frac{np}{2} log 2\pi - \frac{1}{2}\sum_{t=1}^{n}(\log|\boldsymbol{F}_t| + \boldsymbol{v}_t{}'\boldsymbol{F}_t^{-1}\boldsymbol{v}_t)$$

where $\widehat{\boldsymbol{\alpha}}_{t|t-1}$ is the estimate of $\boldsymbol{\alpha}_t$ based on $\boldsymbol{Y}_{t-1}$, $\boldsymbol{F}_t = Var(\boldsymbol{y}_t|\boldsymbol{Y}_{t-1})$ and the number p is the size of the observation vector $\boldsymbol{y}_t$. $\boldsymbol{F}_t$ and $\boldsymbol{v}_t$ are outputs of the Kalman filter equations and thus the loglikelihood is readily calculated.

2.2 Econometric model structures

In addition to the ability of state space models to allow efficient recursive computational operations due to its Markovian nature, a second feature that makes it attractive to econometricians is that it readily lends itself to a structural description of the model. In econometrics, the ability of analysts to specify and validate economic relationships in an intuitive manner is almost as important as the estimation results themselves. This is an attribute in which state space models are significantly superior to the traditional Box-Jenkins models.

Andres Harvey was the first to show how a wide variety of structural econometric models could be cast in state space form (Harvey 1989) and his text presents state space representation of traditional structural models (trend, cyclical, seasonal, calendar components) and describes the straightforward incorporation of explanatory variables, time-varying coefficients, and regime changes. In addition, econometric models are often non-stationary, which is handled in a more straightforward manner than Box-Jenkins models. A traditional univariate structural time series model thus might be specified as in equation (5):

$$y_t = \mu_t + \gamma_t + c_t + \sum_{j=1}^{k} \beta_j x_{jt} + \delta\omega_t + \varepsilon_t \qquad (5)$$

where μ_t is the level with a v_t trend, γ_t is a seasonal component for an s-season model, c_t is a cyclical component, β_j is the regression coefficient associated with explanatory variable x_{jt}, δ is the magnitude of a change in the level of the time series due to an intervention at time t, and ε_t is the error or disturbance term. Harvey showed that this structural time series model could be readily modelled in a state space form (equations (1) and (2)) with state vector:

$$\boldsymbol{\alpha}_t = [\mu_t\ v_t\ \gamma_t\ \gamma_{t-1} \dots \gamma_{t-(s-2)}\ c_t\ c^*{}_t\ \beta_{1t} \cdots \beta_{kt}\delta]' \qquad (6)$$

and coefficient matrices:

$$\boldsymbol{Z}_t = [\boldsymbol{Z}_{[\mu]}\ \boldsymbol{Z}_{[\gamma]}\ \boldsymbol{Z}_{[c]}\ \boldsymbol{Z}_{[\beta]}\ \boldsymbol{Z}_{[\delta]}\ ]$$
$$\boldsymbol{T}_t = diag(\boldsymbol{T}_{[\mu]}, \boldsymbol{T}_{[\gamma]}, \boldsymbol{T}_{[c]}, \boldsymbol{T}_{[\beta]}, \boldsymbol{T}_{[\delta]})$$
$$\boldsymbol{R}_t = diag(\boldsymbol{R}_{[\mu]}, \boldsymbol{R}_{[\gamma]}, \boldsymbol{R}_{[c]}, \boldsymbol{R}_{[\beta]}, \boldsymbol{R}_{[\delta]})$$
$$\boldsymbol{Q}_t = diag(\sigma_\mu^2, \sigma_v^2, \sigma_\gamma^2, \sigma_c^2, \sigma_{c*}^2, \sigma_{\beta 1}^2, \dots \sigma_{\beta k}^2, \sigma_\delta^2)$$
$$H_t = \sigma_\epsilon^2$$

where:

$$\boldsymbol{Z}_{[\mu]} = [1\ 0] \qquad \boldsymbol{Z}_{[\gamma]} = [1\ 0\ \dots 0]$$
$$\boldsymbol{Z}_{[c]} = [1\ 0] \qquad \boldsymbol{Z}_{[\beta]} = [x_{1t} \dots x_{kt}]$$
$$\boldsymbol{Z}_{[\delta]} = \omega_t$$

$$\boldsymbol{T}_{[\mu]} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$
$$\boldsymbol{T}_{[\gamma]} = \begin{bmatrix} -1 & -1 & \cdots & -1 & -1 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & & \ddots & & \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$
$$\boldsymbol{T}_{[c]} = \rho_c \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix}$$
$$\boldsymbol{T}_{[\beta]} = I_k \qquad \boldsymbol{T}_{[\delta]} = \delta$$

$$\boldsymbol{R}_{[\mu]} = I_2 \qquad \boldsymbol{R}_{[\gamma]} = [1\ 0\ \dots 0]'$$
$$\boldsymbol{R}_{[c]} = I_2 \qquad \boldsymbol{R}_{[\beta]} = I_k$$
$$\boldsymbol{R}_{[\delta]} = 1$$

where the cyclical component c_t generally models a business cycle of period $2\pi/\lambda_c$ and a damping factor $0 < \rho_c < 1$.

As summarized by Kitagawa & Gersch, (Kitagawa & Gersch 1996) "The crucial points are that essentially every type of stationary and nonstationary linear and nonlinear time series model can be cast in state space form, and because of the recursive computational properties associated with the Markovianness of the state process, the likelihood of a time series model can be computed in $O(N)$ time ... in contrast with ordinary least squares computations which have $O(N^3)$."

Thus, by the mid-1980s, econometricians lead by Andrew Harvey and James Durbin of the London School of Economics and the University of Cambridge were advocating the innovative use of state space models and Kalman filtering in the modeling of micro- and macro-economic systems. As summarized by Andrew Harvey, "If a model can be put in state space form, a number of powerful statistical results immediately become available … optimal prediction via the Kalman filter, optimal estimates of unobserved components via smoothing, and estimation of unknown model parameters in the model because it enables the likelihood function to be broken down in terms of one-step-ahead prediction errors." (Harvey 1987)