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The performance evaluation of a multi-stage JIT production system with stochastic demand and production capacities

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ABSTRACT

This paper discusses a single-item, multi-stage, serial Just-in-Time (JIT) production system with stochastic demand and production capacities. The JIT production system is modeled as a discrete-time, M/G/1-type Markov chain. A necessary and sufficient condition, or a stability condition, under which the system has a steady-state distribution is derived. A performance evaluation algorithm is then developed using the matrix analytic methods. In numerical examples, the optimal numbers of kanbans are determined by the proposed algorithm. The optimal numbers of kanbans are robust for the variations in production capacity distribution and demand distribution.

1. Introduction

Just-in-Time (JIT) production systems have been created based on the main objective of reducing cost by eliminating waste during the production process. The fundamental property of the system is using the "pull" control mechanism. In JIT production systems, each subsequent process withdraws the necessary parts from a preceding process at the necessary point in time, and the preceding process produces those parts withdrawn by the subsequent process. Kanbans are simple tools used to implement this pull-production control mechanism throughout the system. Among the many studies on the subject, Monden (1993) explains the JIT production systems in detail. According to him, JIT production systems require both input stock in the form of parts and output stock in the form of products at each stage. To maintain these systems, two types of kanbans, or production-ordering kanban and withdrawal kanban, are used as tools to control the production and withdrawal quantities at each stage, respectively.

Much work has been devoted to evaluating the performance of JIT production systems controlled only by production-ordering kanbans. Many authors use simulations to make evaluations (e.g., Ardalan, 1997; Chu and Shin, 1992); however, some authors have developed analytical methods. JIT production systems using only production-ordering kanbans have been evaluated using analytical methods in the following papers. Deleersnyder et al. (1989) analyzed a JIT production system using a discrete-time Markov process: numerical computations were used to study the effects of the number of kanbans, machine reliability, demand variability and safety stock requirements on the performance of the system. Mitra and Mitrani (1990, 1991) studied a multi-stage, serial JIT production system: the subsystem corresponding to each stage was analyzed precisely and an approximation algorithm was devised using a decomposition technique. Wang and Wang (1990) studied multi-item JIT production systems using Markovian queues, and determined the optimal numbers of kanbans for serial, merge- or split-type JIT production systems. Mascolo et al. (1996) used synchronization mechanisms and broke down the original system into a set of subsystems, each of which was analyzed using product-form approximation: an iterative procedure was developed to determine the performance measures of the system. Matta et al. (2005) considered two different kanban release policies (i.e., independent and simultaneous systems), and compared them by approximate analytical methods. Kreig and Kuhn (2004) proposed a decomposition-based analytical evaluation method for a single-stage, multi-product JIT production system with state-dependent setups and lost sales: performance measures of the original system were obtained using an approximation technique.

The JIT production systems using two types of kanbans have been evaluated using analytical methods in the following papers.

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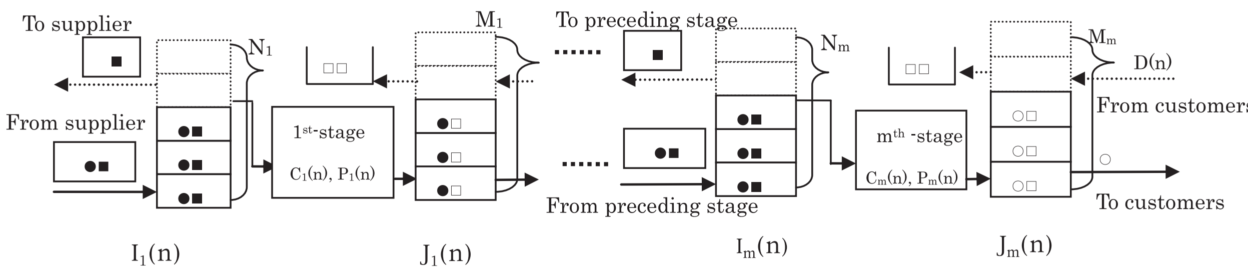

Fig. 1. Multi-stage JIT production system.

Kamarkar and Kerme (1989) constructed Markov models for the JIT production systems and studied the effect of a batch-sizing policy and the number of kanbans on the expected inventory and back-order costs. Ohno et al. (1995) studied a single-stage JIT production system, derived a stability condition for the system and proposed an algorithm for determining the optimal numbers of kanbans using the probability-generating function of the total backlogged demand. Gurgur and Altiok (2004) broke down a JIT production into subsystems using two-node decomposition and obtained an analytical solution for each subsystem: a fixed-point iteration algorithm was proposed for evaluating the performance of the system.

Yang (2000) made a comparison of pull-production control policies using production ordering kanbans and two types of kanbans. As an extension of JIT production systems, Buzacott and Shanthikumar (1993) proposed a wide class of inventory-production systems using four kinds of tags, which they call "PAC systems." PAC systems with specialized parameters can express the base stock, MRP, CONWIP, kanban and other systems. Recently, Kotani (2007) studied a new kanban system with the *e*-Kanban, which utilizes computers and communications networks to transfer information to suppliers, instead of the withdrawal kanban.

This paper discusses a single-item, multi-stage, serial Just-in-Time (JIT) production system using two types of kanbans in which customer demands and production capacities are stochastic. The system is modeled as a discrete-time, M/G/1-type Markov chain with the unit of time being one withdrawal cycle. A necessary and sufficient condition, or a stability condition, under which the JIT production system has a steady-state distribution is derived. A performance evaluation algorithm is then developed using the matrix analytic methods. This paper shows how the matrix analytic methods can be applied for the performance evaluation of the JIT production system under the stability condition. It is therefore essentially an extension of Ohno et al. (1995) to a multi-stage case.

M/G/1-type Markov chains, pioneered by Neuts (1989), provide a framework that enables the exact analysis of frequently encountered classes of queueing models. The matrix analytic methods for M/G/1-type Markov chains are widely used to evaluate communications network systems. One feature of the matrix analytic methods is that accurate numerical solutions can be obtained for problems in which exact solutions are difficult to obtain otherwise. Iwase and Ohno (2008) derived a performance evaluation method for a one-stage, make-to-order production inventory system using the matrix analytic methods for M/G/1-type Markov chains. However, to our knowledge, with the exception of that paper, the matrix analytic methods for M/G/1-type Markov chains have not been applied to evaluate production inventory systems.

This paper is organized as follows: the JIT production system is described in Section 2, the associated M/G/1-type Markov chain is presented in Section 3, the stochastic properties and the stability condition for the Markov chain are derived in Section 4, the cost structure of the system and an algorithm for evaluating system performance are presented in Section 5, and the optimal numbers of kanbans are determined by the proposed algorithm in Section 6.

2. The multi-stage JIT production system

2.1. The kanban system

In the kanban system, production-ordering kanbans are attached to the containers which are periodically withdrawn according to the withdrawal kanbans in the withdrawal kanban post at the subsequent process. When a container is withdrawn, the production-ordering kanban is detached from the container and placed in the production-ordering kanban post. Next, a withdrawal kanban is attached to the container and it is taken to the subsequent process. At the preceding process, the parts are processed according to the ordinal sequence of the production-ordering kanbans in the production-ordering kanban post. Once the first part in a container has been used, the withdrawal kanban is detached from the container and placed into the withdrawal kanban post.

2.2. System description

The JIT production system discussed in this paper is shown in Fig. 1. The system has "m" stages in series, produces a single type of product and there is an infinite supply of raw materials at the input point of the first stage. From upstream-to-downstream, the stages are numbered 1st-stage to mth-stage, and in each stage, two types of kanbans (i.e., withdrawal and production-ordering kanbans) are used. The lead-time for the delivery of the parts, number of production-ordering kanbans and number of withdrawal kanbans in the ith-stage are represented as L_i, M_i and N_i, respectively. It is assumed that the each container's capacity is equal to one.

The discrete-time formulation is adapted: the constant withdrawal cycle is set as one period, and period "n" is set as the interval from time n to immediately before the time $n+1$. For each customer demand arriving in period n, one unit of product is supplied at the beginning of period $n+1$ if there is an inventory of products. The customer demand is backlogged (i.e., backlogged demand) if there is no inventory of products.

The detached production kanbans associated with the parts consumed in the 1st-stage in period n allow parts from an outside supplier to be delivered to the 1st-stage at the beginning of period $n+L_1+1$. For $2 \leqslant i$, the quantity of parts consumed in the ith-stage in period n is delivered to the ith-stage from the $(i-1)$-stage at the beginning of period $n+L_i+1$ or later depending on the upstream state at the beginning of period n. It is assumed that the production capacities and customer demand in each period are independent random variables. The total backlogged demand is set as the sum of backlogged demand and the number of production-ordering kanbans in the mth-stage production-ordering post. The following notations are used: