Constrained multi-objective optimization of storage ring lattices

Riyasat Husain a,b,*, A.D. Ghodke a

a Accelerator Physics Section, Raja Ramanna Centre for Advanced Technology, Indore, 452 013, India
b Homi Bhabha National Institute, Training School Complex, Anushakti Nagar, Mumbai 400 094, India

1. Introduction

The lattice optimization for the low emittance synchrotron light sources is an involved task, which belongs to a class of multi-objective optimization problem. One needs to optimize various objectives simultaneously [1]. For a synchrotron light source, its brightness is the main figure of merit. It is inversely proportional to the product of horizontal and vertical beam emittances. The beam emittance [2] is a complicated function of the lattice properties driven by the distribution of dipole and quadrupole magnets and their strengths. Minimizing the beam emittance is not the only objective of the lattice design, instead one also needs to optimize for the operational feasibility, for accommodating larger number of insertion devices, for achieving good injection efficiency and sufficient beam lifetime etc. The sextupole magnets with high strengths are the integrated elements of the low emittance lattices to correct for high natural chromaticities, which may lead to shrinking of dynamic aperture (DA). Keeping in view that all types of magnets are to be used, low emittance and large DA are the ultimate optimization goals. In some machines, there is a requirement of getting special distribution of lattice functions, for example, low beta is required for minimizing the effects of insertion devices in operation and high horizontal beta is required for the conventional injection in storage rings. The smaller momentum compaction factor is desirable for producing the shorter bunches. The optimization of the distribution of the lattice functions and still minimizing the beam emittance and increasing DA are some of the conflicting requirements.

Earlier, various traditional approaches were applied for optimizing the lattice parameters of the storage rings. In these approaches, one needs to first find a stable solution, often by trial and error, that roughly meets the desired properties, and then goal is to locally improve around it. A technique called GLASS (GLocal scan of All Stable Settings) [3] was proposed in which the properties of the lattices are systematically calculated for all stable solutions. However, because the computing time is exponentially large, depending on the numbers of variables and step sizes, GLASS currently is unfeasible for problems with more than 4–5 variables [4]. Usually, the variables that govern such problems are related to one another in a very complicated way, and finding the best combination of them can be a challenging task. Normally, the relations between these decision variables can be translated into objective functions, and their values can be interpreted as a measure of the quality offered by that particular combination of the decision variables to a particular aspect of the solution. The n-dimensional optimization problems with m individual objectives can be stated as [5]:

\[
\text{Minimize/Maximize : } F(x) = [f_1(x), f_2(x), f_3(x), \ldots, f_m(x)]; \quad (1)
\]
where, $\mathbf{x}$ is a vector of $n$ dimensional decision variables $\mathbf{x} = (x_1, x_2, x_3, \ldots, x_n)$. Here, $x_i^{(L)}$ and $x_i^{(U)}$ are the lower and upper bounds of the decision variables $x_i$. The functions $f_i(\mathbf{x})$ through $f_n(\mathbf{x})$ indicate the individual objective functions. In almost all practical cases, finding good solutions is not a problem, but finding the best solution is much more involved. Moreover, for most problems encountered in practice, the objective functions $f_i(\mathbf{x})$ through $f_n(\mathbf{x})$ are highly nonlinear, non-differentiable, or have no way of determining initial estimates close to the global optimum. The practical approach to tackle such problems is to use the meta-heuristic optimizers, which use a population of trial solutions, and apply certain probabilistic rules to generate a new population, which converge to the global minimum of the objective functions with high probability. Over the years, many such algorithms have been developed, of which the Genetic Algorithm (GA) [1,5–10], Particle Swarm Optimization (PSO) [11–13] are most widely used in optimization of accelerator parameters.

GA is a class of evolutionary algorithms of which Non-dominated Sorting Genetic Algorithm-II (NSGA-II) [5] is more popular, which is based on natural evolution of crossover, mutation and selection. The PSO is a class of swarm intelligence algorithm. PSO follows the self-organizing behaviour of social animal living in a group. It has been recently used to optimize the Linac operation and nonlinear dynamics of storage rings [11–13].

It has been demonstrated that both the algorithms—GA and PSO are powerful and effective in solving the problems for highly nonlinear objectives, which have a large number of local optima. Comparative studies of these two techniques have been performed in Ref. [13] for optimizing the momentum and dynamic apertures of SPEAR3 storage ring. Two cases of initial population generation were considered: (i) purely random distribution within lower and upper bound, and (ii) initial population seeded with the nominal good solution and a small random distribution around it. It was reported that PSO converges faster than GA, and is not dependent on the distribution of initial population, while GA depends on initial population distribution considerably. In order to check this further, a performance comparison of these two algorithms by applying them to a problem with two objective functions—beam emittance and dispersion function at the centre of straight section are performed for the case of Indus-2. It was found that if initial population are randomly generated, then PSO and GA give almost equivalent results, however a diversity in the variable space is adequate in case of PSO.

Some authors have applied another class of evolutionary algorithm, called Differential Evolution (DE) [14], which is based on the globalized pseudo-derivatives. DE is a very powerful and simple population based algorithm like genetic algorithms using the similar operations—crossover, mutation and selection. The main difference in constructing the better solutions is that the genetic algorithms rely on crossover, while DE relies on mutation operation. This main operation is based on the differences of randomly sampled pairs of solutions in the population. The algorithm uses mutation operation as a search mechanism and selection operation to direct the search towards the prospective regions in the search space. The DE algorithm also uses a non-uniform crossover that can take the child vector parameters from one parent more frequently than it does from others. DE is an additional recipe for optimization to achieve low momentum compaction factor together with lower beam emittance and dispersion function in the straight section is presented. This uses strengths of all five quadrupole families as the variables. The results obtained by GLASS scan are also presented. The outcome of the study is that DE produces best Pareto optimal front compared to other algorithms in almost equal computation time. The optimization to achieve low momentum compaction factor together with lower beam emittance are discussed in Section 4. In Section 5, we extend the study to explore the alternate low and high beta lattices for commissioning of insertion devices (IDs) and satisfying the off axis injection. In this mode, DE was used for simultaneous optimization of the beam emittance and DA, including two sextupole family strengths as additional two variables. Lastly, in Section 6, summary and future possibilities of applying the DE for upgrade of Indus-2 lattice are presented.

2. Indus-2 lattice

Indus-2 [18,19] is a 3rd generation synchrotron light source located at Raja Ramanna Centre for Advanced Technology (RRCAT) Indore, India, which is optimized for the generation of photons at a critical wavelength of 2 Å. The ring is ~172.5 m in circumference and consists of eight super-periods. The lattice structure of one super-period is a double bend achromat of the expanded Chasman Green type. The lattice functions together with arrangement of magnetic elements in a super-period are shown in Fig. 1.

The DE algorithm uses the mutation operation based on scaled differences of the parent solutions to generate the next generation candidates or trial parameter vectors for optimization. Originally, Storn and Price proposed five different mutation processes or strategies in Ref. [14]. For the present study, we have selected a ‘rand-to-best’ strategy [16,17], which gives a balance between fast convergence and robustness of the algorithm. In this strategy, a trial parameter vector $\mathbf{v}_{ij}$ at generation $j$ is generated according to

$$v_{ij} = x_{ij} + F_1(x_{ib} - x_{ij}) + F_2(x_{ja} - x_{ij})$$

where, $r_1$ and $r_2$ are integer indices chosen randomly from the interval [1, NP] and are different from the running index $i$. $x_{ib}$ is the best solution vector among $N_P$ population members at the generation $j$. $F_1$ is a weight factor for the combination between best and current parent vector, and $F_2$ is a scaling factor, which controls the amplification of the differential vectors and is known as mutation-weighting factor. Too large values of $F_1$ and $F_2$ provide diversity in the solution space and too small values results in fast convergence. Normally there values are in the range 0–2.

To further increase the diversity in solution space, a crossover operation between the target vector $x_{ij}$ and mutant vector generate $v_{ij}$ at generation $j$, is performed. This operation combines the two vectors into a new trial vector $u_{ij}$ as per the following rule:

$$u_{ij} = \begin{cases} x_{ij}, & \text{if } \text{rand}(j) \geq CR, \\ v_{ij}, & \text{otherwise,} \end{cases}$$

where, rand($j$) is a uniform random number between 0 and 1, and $CR$ is the crossover probability. $CR$, $F_1$ and $F_2$ are three important controlling parameters of the algorithm. The newly generated trial solution $u_{ij}$ is checked against the parent solution $x_{ij}$ through the selection operations like in NSGA-II.

In this paper, all three multi-objective optimization algorithms—GA, PSO and DE are used to compare their performance for the case of linear lattice optimization of Indus-2 storage ring. The Indus-2 lattice is described in Section 2. In Section 3, a study for the optimization of beam emittance and the dispersion function in the straight section is presented. This uses strengths of all five quadrupole families as the variables. The results obtained by GLASS scan are also presented. The outcome of the study is that DE produces best Pareto optimal front compared to other algorithms in almost equal computation time. The optimization to achieve low momentum compaction factor together with lower beam emittance and dispersion function in the straight section is presented. This uses strengths of all five quadrupole families as the variables. The results obtained by GLASS scan are also presented.
دریافت فوری متن کامل مقاله

امکان دانلود نسخه تمام متن مقالات انگلیسی
امکان دانلود نسخه ترجمه شده مقالات
پذیرش سفارش ترجمه تخصصی
امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
امکان دانلود رایگان ۲ صفحه اول هر مقاله
امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
دانلود فوری مقاله پس از پرداخت آنلاین
پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات