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A just-in-time three-level integrated manufacturing system for linearly time-varying demand process

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ABSTRACT

This paper considers a just-in-time (JIT) manufacturing system in which a single manufacturer procures raw materials from a single supplier, process them to produce finished products, and then deliver the products to a single-buyer. The customer demand rate is assumed to be linearly decreasing time-varying. In the JIT system, in order to minimize the suppliers as well as the buyers holding costs, the supply of raw materials and the delivery of finished products are made in small quantities. In this case, both the supply and the delivery may require multiple installments for a single production lot. We develop a mathematical model for this problem, propose a simple methodology for solving the model, and illustrate the effectiveness of the method with numerical examples.

1. Introduction

Over the last few decades, the supply chain design and management issues have been widely studied. Still, these are attractive research topics, partly because of the relentless drive to lower cost and partly because of improving service quality through efficient information sharing/exchange among different parties involved in the entire supply chain. Since 1980, there have been numerous studies discussing implementation of JIT and its effectiveness in US manufacturing system [1]. In the JIT integrated manufacturing system, the raw material supplier, the manufacturer, and the buyer work in a cooperative manner to synchronize JIT purchasing and selling in small lot sizes as a means of minimizing the total supply chain cost.

Goyal [2] was probably one of the first to introduce the idea of integrating a single vendor with a single buyer as part of a simple two entity supply line system. Later, Banerjee [3] developed a model where the vendor manufacture at a finite rate and that follows a lot-for-lot policy. Since then, many variants are reported in the literature. For example, the two-level vendor–buyer problem and its related issues can be found in Goyal [4], Hill [5,6], Valentini and Zavanella [7], Zanoni and Grubbstrom [8] and Hill and Omar [9]. The manufacturing-vendor model can be found in Sarker and Khan [10]. Omar and Smith [11] and Omar [12] extended both models by considering time-varying demand rate. Recently, Benkherouf and Omar [13] extended the model presented in Omar [12] further and proposed a procedure for finding an optimal solution analytically.

For a three stage supply chain, Banerjee and Kim [14] developed an integrated JIT inventory model where the demand rate, production rate, and delivery time are constant and deterministic. Munson and Rosenblatt [15], Lee [16], Lee and Moon [17], and Jaber and Goyal [18] also considered a similar problem consisting of a single raw material supplier, a single vendor, and a single buyer. Jaber et al. [19] extended the work of Munson and Rosenblatt [15] by considering ordering quantity and price as decision variables.

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None of the above researchers consider a collaborative three stage supplier–manufacturer–buyer inventory system with time-varying demand rate. In this paper, by assuming linearly decreasing time-varying demand rate at the buyer's end, we develop a mathematical model for a coordinated three-level JIT supplier-manufacturer-buyer inventory system, propose a simple solution approach for solving the model, and illustrate the effectiveness of the method by numerical examples.

The paper is organized as follows. In Section 2, we summarize the assumptions made to define the problem and the notations required to develop the mathematical model. In Section 3, we formulate the problem. The numerical results and analysis are presented in Section 4. Finally we draw our conclusions in Section 5.

2. Assumptions and notations

The supply chain problem considered here consists of a single raw material supplier, a single manufacturer who manufactures the finished product in batches at a finite rate and delivers it at equal shipment size or at equal replenishment interval to a single buyer. At the beginning of the production cycle, the inventory at the manufacturer's end is assumed to be zero. However, the inventory level at the buyer's end is just enough to satisfy their demand until the next delivery arrives. The stock value normally increases as a product moves down the distribution chain, and therefore the associated holding costs are considered to be higher. However, as indicated earlier, we want as little stock as possible at the buyer's side. So the optimal policy is to order when the buyer is just about to run out of stock.

The total cost for this system includes all costs from both buyer and manufacturer. The buyer's cost consists of shipment and holding cost. The manufacturer's cost includes setup and holding cost of finished products, and ordering and holding cost of raw materials.

To develop a JIT three-level integrated inventory model, we have made the following assumptions.

1. No shortages are permitted.
2. A single product inventory system is considered over a finite planning horizon.
3. During the production up-time, the finished product becomes immediately available to meet the demand.
4. The demand rate of finished product at any time t in $(0, T)$ is $f(t)$ and assumed to be linearly decreasing.
5. The finite production rate is P units per unit time where $P > f(t)$ for all t.
6. Only one type of raw material is considered.

The following notations are used in modeling the problem.

1. C_p is the manufacturing set-up cost.
2. C_1 is the ordering cost for raw material 1.
3. C_b is the shipment cost
4. H_p is the inventory carrying cost per unit per unit time for finished product at the manufacturer end.
5. H_1 is the inventory carrying cost per unit per unit time for raw material 1.
6. H_b is the inventory carrying cost per unit per unit time at the buyer's side.
7. n is the number of shipment.
8. m is the raw material's lot size factor and equivalent to the number of raw material installment.

3. Mathematical formulation

To show the relative lot sizing in three stages, the inventory levels with time plot, where $m = 3$ and $n = 4$, is depicted as Fig. 1. The bottom part of the figure shows the inventory level of raw material. The raw material required for each production batch is delivered in equal quantity in m cycles. Similarly, one production batch is delivered in n small equal lots to the buyer as shown in the top part of the figure. The middle part shows the inventory level for the production system.

3.1. Buyer's total cost

For convenience of formulation, the amount of stock holding at the buyer's end during the time period T is calculated assuming that either all remaining x units from the previous cycle is fully consumed or $t_0 = 0$. From Fig. 1, the buyer stock level, IB_{i+1} in any period $i+1,\ i = 0, 1, \dots, n-1$ is

$$IB_{i+1} = q - \int_{t_i}^{t} f(t)dt = \int_{t_i}^{t_{i+1}} f(t)dt - \int_{t_i}^{t} f(t)dt = \int_{t}^{t_{i+1}} f(t)dt.$$

The total time weighted stock holding, TWB, from t_0 to t_n is given by

$$TWB = \sum_{i=0}^{n-1} \int_{t_i}^{t_{i+1}} IB_{i+1} dt,$$

where (t_i, t_{i+1}) is the cycle for $(i+1)$th shipment. In this paper we consider two possible policies. In the first policy we assume equal shipments interval where $t_i = i(T/n)$. For the second policy we assume equal shipments lot size where for