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Stylized facts in internal rates of return on stock index and its derivative transactions

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Abstract

Universal features in stock markets and their derivative markets are studied by means of probability distributions in internal rates of return on buy and sell transaction pairs. Unlike the stylized facts in normalized log returns, the probability distributions for such single asset encounters incorporate the time factor by means of the internal rate of return, defined as the continuous compound interest. Resulting stylized facts are shown in the probability distributions derived from the daily series of TOPIX, S & P 500 and FTSE 100 index close values. The application of the above analysis to minute-tick data of NIKKEI 225 and its futures market, respectively, reveals an interesting difference in the behavior of the two probability distributions, in case a threshold on the minimal duration of the long position is imposed. It is therefore suggested that the probability distributions of the internal rates of return could be used for causality mining between the underlying and derivative stock markets. The highly specific discrete spectrum, which results from noise trader strategies as opposed to the smooth distributions observed for fundamentalist strategies in single encounter transactions may be useful in deducing the type of investment strategy from trading revenues of small portfolio investors.

Keywords: Universal features; Stylized facts; Returns on strategy; Trend investment; Technical analysis; NIKKEI 225 derivative market

1. Introduction

Stylized facts in market indicator data [1–7] have been reported, up to date, especially on the session-to-session basis, such as the fat tail law in the distribution of log-normalized returns $R_t \equiv \log(P_i/P_{i-1})$ on various time scales and aggregation levels for numerical data P_i at time ticks $i = 1, \ldots, N$. The reasons for introducing this form of statistics are well known: the values P_{i-1} and P_i should a priori correlate most strongly; the relative factor P_i/P_{i-1} is suitable for the analysis of market trends; and the application of the log function brings additional symmetry between the cases of indicator increase and decrease, while preserving the original value of the relative factor for small market changes, $\log(1+x) \sim x$ for $|x| \ll 1$. Finally, it is well established that the histograms of R values are practically symmetric for long-term data series with respect to the inversion of $R \to -R$, and may have fat tails (relative to the normal distribution), which are known as the stylized facts. The above factors constitute a strong case for studying the indicator data in terms of probability distributions

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of normalized log returns. Last but not least, the ratio $P_i/P_{i-1} - 1$ is the profit (or loss) from buying the asset at session (or tick) $i-1$ and selling it at i without transaction costs.

In this paper, we follow and extend the interpretation of R_t as an elementary transaction (i.e., a function on tick data) as a means of market data analysis. While the standard form of this function, i.e., $P_i/P_j - 1$ is based only on $i-j=1$ correlations and can be both positive and negative, our intent is to develop a more general criterion which can allow for complex correlations between all ticks, i.e., $j<i$ within the entire data sample. In particular, the size of the tick segment $i-j$ in our framework *depends* on the analyzed data, and is defined by taking j as the minimum of $i'>i$, for which a required *internal rate of return* (IRR) is achieved on the tick data pair R_i and R_j. In broader terms, by using a probability density of returns on fundamental transactions, it may be possible to relate market stylized facts (data) to the ultimate paradigm of market investment, the optimal transaction (strategy). The problem of optimal investment [8] is also central in mathematical finance.

Both the standard distribution of normalized log returns, and the above motivated distribution of internal rates of return (IRR transform) result from elementary transactions on a risky asset, hereafter referred to as a single encounter. The optimal transaction may involve a two-component portfolio (money and a stock index in this paper) or a multi-component portfolio with particular stock titles. Depending on whether the transaction profit is reinvested or not, the geometric mean or the arithmetic mean of the expected return of single encounter investment should be optimized [9]. The former case of geometric mean is known as the Kelly criterion [10], which specifies the optimal single encounter ratio x for one risky asset with return b and probability p by $x = p - (1-p)/b$, in order to maximize the expectation value of the logarithm of the single encounter outcome, i.e., the economics' utility function on one hand, which is identical with the entropy of this system on the other hand. That is where the present, probabilistic formulation of single asset encounter naturally relates to the Shannon's information entropy, and its generalized models.

In general, the present approach should complement a variety of econometric, econophysics, statistical and artificial intelligence approaches to time series analysis [11–14].

The paper is organized as follows. Section 2 explains the theoretical rationale of the generalized stylized features as a transform in the internal rates of return. The single encounter investment in the continuous interest model is applied to the time series of TOPIX, S & P 500 and FTSE 100 data, for which the resulting IRR transforms are derived and their stylized features are discussed. In Section 3, the characteristic spectrum of one particular single encounter strategy based on the comparison of short-term and long-term trends is shown. Section 4 demonstrates both the standard and the present formulations of the stylized facts on the minute-tick data from NIKKEI 225 index, and its futures market, and discusses their causal relation in terms of the IRR transform. In Section 5, the stylized facts in the IRR transform are distinguished from the standard formulation in terms of normalized log returns. We conclude with final remarks in Section 6.

2. Internal rates of return—market transform

As mentioned above, the stylized facts in indicator time series [3–7] are usually studied by means of log-normalized returns, $\log(P_i/P_{i-1})$. Let us consider an investment on stock market index (which itself is one of the fundamental types of portfolios) for a variable period of t sessions (or ticks) with a minimal required profit rate r per session. In other words,

$$0 = C_0 - \frac{C_t}{(1+r)^t} \tag{1}$$

with an unknown period t, cash flow C_0 equal to the buying price, and cash flow C_t equal to the selling price. The termination condition for the investment thus requires

$$P_t \geqslant P_0(1+r)^t \tag{2}$$

at the smallest possible time t for which the above inequality holds. In economics terms, this can correspond to the maximal implicit interest rate r of reverse repo operation in the interval $\langle 0, t \rangle$. The fixed value of r will be the basis of the IRR transform.[1]

[1] Although the fixed r-value is a mathematical concept here, it also corresponds to the fundamentalist strategy.