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A geometric representation of profits in a supply chain network

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A R T I C L E I N F O

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A B S T R A C T

Economists express profits as areas representing producer's surplus or consumer's surplus corresponding to a pair of supply and demand functions. A similar representation can be employed in a supply chain network where there can be several producers/suppliers and several consumers/retailers in various trading situations. We aim to graphically represent the profits corresponding to a combination of the decisions of all participants in a supply chain, simultaneously, as areas of non-overlapping regions on the same graph. This way, the shares of the participants in the total can be visually observed and the interdependencies and the inefficiencies in the chain can be detected where the decisions and the corresponding profits are generally interdependent. Such a visual tool can be used in evaluating as well as in designing a supply chain.

1. Introduction

Economists express profits as areas representing producer's surplus or consumer's surplus corresponding to a pair of supply and demand functions. A similar representation can be employed in a supply chain network where there can be several producers/suppliers and several consumers/retailers in various trading situations over a single period. A retailer and all suppliers contributing to the production of the item sold by that retailer share the profit created by the final demand of that retailer. We use this fact to graphically represent the profits corresponding to a combination of the decisions of the participants in a supply chain, simultaneously, as areas of non-overlapping regions on the same graph. This methodology is the main contribution of the present study. It also provides geometric conditions for equilibrium. Using this methodology, the profits can be visually observed; the interdependencies and the inefficiencies in the chain can be detected by inspection. This representation can be used in evaluating as well as in designing a supply chain as a complementary visual tool to mathematical analysis. It can be used in teaching courses related to supply chain as well as in situations for which visual impact is more suitable.

The tasks performed on a party of a product can be considered as a network. Hence, the suggested representation can also be used for visualizing the value/cost stream generated by the tasks within a firm.

The supply chain configuration is important for this representation. Although it is straight forward to represent these profits as areas in two-dimensional plane, our approach is general enough to be applied in different chain configurations: a vertical chain starting with a supplier ending with a retailer; a retailer and his suppliers; a lateral chain consisting of a supplier and his retailers; and finally a supply chain network. In each case, we try to characterize the equilibrium decisions of all parties geometrically, define the areas of non-overlapping regions corresponding to the participants' profits.

In the present work, it is not our purpose to find the equilibrium decisions in different supply chain settings; for most of the network configurations considered here, equilibrium decisions have been studied already. The single retailer multiple capacitated supplier chain equilibrium is worked in relatively more detail since its setting is different than those in the literature. A comprehensive review of the supply chain models can be found in Simchi-Levi et al. (2004). Among many others, Lariviere and Porteus (2001) consider equilibrium with single retailer and single supplier and the factors effecting it; Li (2008) and Shang and Song (2007) work on the equilibrium decisions in a vertical supply chain; Chen et al. (2001), Geng and Mallik (2007), Wu et al. (2012) and Chan and Lee (2012) consider single supplier and multiple retailers; Seshadri et al. (1991), Chen et al. (2001), Iyengar and Anuj (2008), Qi (2007) and Glock (2012) consider single buyer and multiple suppliers; Ganeshan (1999) and Seshadri et al. (1991) consider a set of suppliers selling to a set of retailers. They analyze profit maximizing decisions. Cachon and Lariviere (2005) use graphs on which the profits of a single retailer and a single supplier are areas of non-overlapping regions. The present work generalizes the graph to the profits in a supply chain network. We aim (i) to give a representation for any combination of decisions, (ii) to characterize the equilibrium decisions geometrically if possible and (iii) to compare them in terms of this representation.

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Profits under different settings are usually compared using their magnitudes that depend on participants' decisions as well as the uncontrollable parameters. For example, if a retailer profits more than another, this may be due to his higher selling price rather than his better decisions. In the proposed representation, the profits are necessarily scaled by appropriate market price. As a result, "the market price effect" is reduced in comparisons. Supply chain performance is measured in many dimensions. Gunasekaran et al. (2007) give a general framework detailed in these dimensions including cost and flexibility considerations. We propose a simple efficiency measure in terms of the "scaled" profits in a decentralized chain with respect to the centralized chain to capture the losses due to decentralization, chain configuration, inefficient costs as well as loses due to arbitrary decisions. It shows the percentage of the total available profit that can be created in the market captured by the chain. This measure is straight forward for single retailer chains because it is simply the ratio of the profit of decentralized chain to the profit of centralized chain. When there are several retailers with different market prices in a supply chain it is not equal to that profit ratio; scaling the profits with the corresponding prices it provides a means to aggregate chain profits and to measure efficiency eliminating the market price effect. This measure improves as the chain becomes more "compact"/centralized.

We assumed random market demand of the chains however the analysis follows with any demand that is a function of price.

2. Single retailer chains

In this section, different chain configurations with single retailer are considered.

2.1. A single retailer-single supplier chain

Single period inventory problem, the newsvendor problem, is studied widely in many studies with various extensions. The random market demand for the retailer's product is X with pdf $f(x)$ and an invertible cdf $F(x)$ and $0<E(X)<\infty$. The expected profit of the retailer $P(Q)$ and the profit of the supplier $S(w)$ are

$$P(Q)=rE[\min\{X,Q\}]-wQ$$

$$S(w)=(w-c)Q(w) \tag{1}$$

where r is the selling price of the retailer, w is the selling price of the supplier and c is the unit production cost of the supplier. The retailer maximizes $P(Q)$ with respect to Q and the supplier maximizes $S(w)$ with respect to w. Problems of determining the order quantity Q and supplier selling price w are analyzed in Lariviere and Porteus (2001) investigating the effects of market size, the demand distribution parameters. Here we use this case as a starting point of the geometric representation.

The profits in (1) can be represented as integrals and they correspond to areas of non-overlapping regions in the plane. The following proposition defines these regions first for arbitrary w and Q and then for equilibrium w and Q as demonstrated in Fig. 1.

Proposition 1. (i) *Given $c\le w\le r$ and Q, let $w_Q=r(1-F(Q))$, $P=P(Q)$ and $S=S(w)$. Then S/r is the area of the rectangle defined by $x=(r-w)/r$, $x=(r-c)/r$, $y=0$ and $y=Q$; P/r is the sum of the areas of the region defined by $x=0$, $x=(r-w_Q)/r$, $y=0$ and $y=F^{-1}(x)$ and of the region defined by $x=(r-w_Q)/r$, $x=(r-w)/r$, $y=0$ and $y=Q$ on the (x,y) plane.*

(ii) *Given r and w as in (i), if the retailer maximizes his expected profit then (i) holds for $Q=Q^*=F^{-1}((r-w)/r)$.*

(iii) *Given r, if the retailer maximizes his expected profit and if the supplier maximizes his profit then (i) holds for the optimal order quantity Q^* and the optimal supplier price w^* and they satisfy*

$$Q^*=F^{-1}\left(\frac{r-w^*}{r}\right)$$

$$f(Q^*)=\frac{w^*-c}{rQ^*} \tag{2}$$

Proofs are given in the Appendix. Fig. 1a, b and c presents examples for (i), (ii) and (iii) cases in the proposition, respectively. In case of single retailer and single supplier, these profit regions correspond to the producer and the consumer surplus in economics. The following notes can provide some more insight.

(i) Since we have a single retailer with demand function $F^{-1}(x)$, the regions can be defined on either $(x,F^{-1}(x))$ or $(F(y),y)$ planes. We prefer to use $(x,F^{-1}(x))$ plane because, in case of more than one retailer, the total demand function can easily be represented by the sum of inverse cdf's rather than cdf's.

(ii) The function $F^{-1}(x)$ is a demand function of the retailer with a random demand that implies the scaling factor $(r-w^*)/r$. This function can be replaced any demand function in r.

(iii) The supplier should give a price w to the retailer to maximize his profit; that is, he tries to fit the maximum area rectangle to the area below $F^{-1}(x)$. The first-order optimality condition in (2) for the supplier can be restated as the diagonal of the rectangle that gives the optimal w, that is, the line connecting $((r-w)/r,0)$ and $((r-c)/r,Q)$ must be parallel to the tangent line to $F^{-1}(x)$ at $x=(r-w)/r$. This condition is satisfied in Fig. 1c while it is not satisfied in Fig. 1a and b.

All the notes (i)–(iii) apply to the following more general cases also. As a result of Proposition 1, the total profit of the chain is bounded by the total area below $F^{-1}(x)$. The following corollary gives this upper bound.

Corollary 1. $P+S\le r\int_0^{(r-c)/r}F^{-1}(x)\,dx$.

The right-hand side is independent of the decisions and is maximum when $c=0$ with $P+S=rE[X]$. The corollary states that for general $0<c\le r$, the maximum profit of the chain is obtained when the chain is centralized with $w=c$. So, we define an efficiency measure as the fraction of the available area below by $F^{-1}(x)$ captured by the retailer's and the supplier's profits as

$$E=\frac{P+S}{r\int_0^{(r-c)/r}F^{-1}(x)\,dx}$$

that is the proportion of the centralized profit captured in a decentralized supplier–retailer chain. The optimal behavior of the participants creates inefficiency in that sense. For a given w the optimal decision of the retailer, as in Fig. 1b, always produces more efficient profits than his suboptimal decisions as in Fig. 1a. As w decreases, the setting approaches to the centralized case. The supplier prices $w<w^*$ as in Fig. 1b produce more and $w>w^*$ produce less efficient profits than the equilibrium decisions as in Fig. 1c.

We aim to generalize the representation of profits on the same plane and this simple efficiency measure to multiple participant mixed structure chains.

2.2. Vertical generalization: an n-stage chain

Consider an n-stage chain with a supplier at stage 1 and a retailer at stage n. Remaining $n-2$ stages represent the other suppliers who buy from the lower supplier and sell to the upper stage. Supplier i buys Q from supplier $i-1$ at a price of w_{i-1}, spend a fixed amount c_i per item and sell Q to the supplier $i+1$ at price