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Multiclass first-order simulation model to explain non-linear traffic phenomena

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Abstract

With the first-order traffic model of Lighthill, Whitham and Richard (LWR), many simple traffic problems can be represented analytically such as a shock formation. However, the LWR model has some deficiencies. For example, among the other things, it fails to replicate interesting non-linear phenomena such as hysteresis and capacity drop as well as the dispersion of traffic platoon when there exists a distribution of desired speeds in heterogeneous traffic. To this end, in this paper, we propose a novel multiclass first-order simulation model based on an approximation of Riemann solver. In the developed model, each vehicle class is only characterized by their desired speeds in a free-flow traffic state where overtaking is allowed. However, when traffic is congested, all vehicle classes must travel at the same *congested* speed and overtaking is not possible. Numerical results show that the proposed model is not only more accurate and reliable than the existing models but also able to explain non-linear traffic phenomena on freeways.

Keywords: Traffic flow; Multiclass; First-order model; Non-linear traffic phenomena

1. Introduction

In recent years, traffic congestion has become one of the main societal issues in many countries in the world. To this end, a clear understanding of the causes of congestion, propagation of traffic congestion and so on is essential in order to support traffic management strategies. Research on traffic flow modelling has been performed since the 1950s, for example by Lighthill and Whitham [1], and Richards [2]—hereafter denoted as LWR—who independently developed a mathematical model for traffic flow operations on freeways based on the similarity between traffic flow and fluid dynamics. Since then mathematical modelling of traffic flow has been an interesting subject of many scientists.

In principle, there are three types of traffic flow models: microscopic models, mesoscopic models and macroscopic models. Microscopic models describe traffic flow at high level of detail such as the movement of individual vehicles, whereas macroscopic models represent traffic flow at low level of detail by aggregate traffic variables such as flow, mean speed and density. Mesoscopic models, on the other hand, deal with traffic flow through probabilistic terms. That is, traffic flow is described by the behaviour of a group of vehicles. An

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example of mesoscopic models is the gas-kinetic model which is used to derive the macroscopic models based on the method of moments [3,4]. In this paper, we mainly focus on the macroscopic modelling approach.

The first-order macroscopic LWR-type models are simple and allow one to represent a formation of shock wave, but its assumption of a steady-state speed density relationship does not allow fluctuations around the so-called *equilibrium* fundamental diagram. Furthermore, some non-linear traffic phenomena such as hysteresis and capacity drop are not represented by the LWR-type models. In real traffic, these phenomena have been observed, e.g. Ref. [5], which indicate that the flow downstream of a jam is smaller than that upstream of the jam. To remedy such deficiencies of the LWR-type models, a lot of research has been devoted to develop higher order (macroscopic) models which can describe the non-linear instabilities of traffic flow. Examples of higher order macroscopic models can be found in Refs. [4,6–14]. Although the higher order models have been able to show some significant improvements over the first-order models in replicating the transitions between traffic congested states and non-linear phenomena such as platoon dispersion, hysteresis and capacity drop (see Refs. [11,15–17]), their higher number of parameters have limited their real-life applications.

Recently, many efforts have been undertaken to extend the original LWR model in order to describe the non-linear traffic flow phenomena. In order to include traffic hysteresis, different fundamental diagrams have been employed in the LWR-type models [18]. This approach is able to describe the transition from one speed–density relationship to another. Some other suggestions have been made in taking into account the interaction between vehicle classes in traffic flow [19]. Along these lines, research has been conducted to model the asymmetric characteristics of the mixed traffic flow based on LWR model [20–23]. However, general applicable models which adequately describe multiclass traffic operations have so far not been well-established. For this reason, we propose in this paper a generalized multiclass first-order simulation model that produces relatively small smoothing effect, is able to replicate the dispersion of traffic platoon and to explain the aforementioned non-linear traffic phenomena such as hysteresis and capacity drop phenomena properly.

This paper is organized as follows. Section 2 illustrates the governing equations for the multiclass first-order traffic model. In this section, we also analyse the important mathematical properties of the model. In Section 3, we propose a multiclass first-order simulation model based on an approximation of the Riemann solver. Some properties of the newly developed model are briefly discussed analytically and numerically in this section. Section 4 illustrates the properties of the model in explaining non-linear traffic phenomena such as hysteresis and capacity drop through a simulation using real data collected on the M25 freeway in England. Finally, we conclude the paper in Section 5.

2. Governing equations for multilcass first-order model

The extension of LWR model to a number of vehicle classes has been carried out recently in Refs. [20,22–24]. In this section, we will first recall the concept of the multiclass LWR model. Let u ($u \in U$) denote the vehicle class index. Let $r^u(x,t)$, $V^u(x,t)$ and $q^u(x,t)$ denote the class specific density, mean speed and flow, respectively. From the conservation law, each vehicle class should satisfy the following equation:

$$\frac{\partial r^u}{\partial t} + \frac{\partial q^u}{\partial x} = 0 \quad \forall u \in U. \tag{1}$$

Eq. (1) can be written in the vector form

$$\frac{\partial \mathbf{r}}{\partial t} + \frac{\partial \mathbf{q}}{\partial x} = 0, \tag{2}$$

where

$$\mathbf{r} = \begin{bmatrix} r^1 \\ r^2 \\ \vdots \\ r^U \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} q^1 \\ q^2 \\ \vdots \\ q^U \end{bmatrix}. \tag{3}$$