Expert Systems with Applications 37 (2010) 8365–8370

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

The cutoff transaction size and Gauss cost functions to the information value applying to the newsboy model

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ARTICLE INFO

Keywords:
Information value
Cutoff transaction size
Newsboy model
Gauss function

ABSTRACT

In this paper, two models are considered which differ in terms of information completeness. The first model involves the retailer having incomplete information regarding the state of customers demand. The second model involves the retailer having full information on the state of customers demand. Step function (here called Gauss function) can be applied such as the cost for mailing letters or packages in the post office and for shipping goods in containers. Therefore, the holding and penalty costs are represented by the Gauss functions to fit in with these practical situations. In addition, we assume that customers with an order larger than a prespecified quantity (here called cutoff transaction size) are still assumed to be satisfied in an alternative way, against additional cost. Moreover, when the maximum demand is large, much more time may be required to determine the optimal solution. Thus, we adopt and modify the algorithm of the Golden Section Search Technique to determine the optimal order-up-to level S and the cutoff transaction size q systematically and provide illustrative numerical example.

1. Introduction

Enormous quantities of information currently are being exchanged between manufactures and retailers, retailers and consumers, companies and investors, and also among parties sharing the same level of a vertical chain. The above situation is particularly true for customized products, and occurs most commonly between retailers and larger customers. Industrial retailer-customer relations recently have changed radically. Consequently, information value has recaptured the interest of academics and practitioners.

A message is valuable only when it can enable a decision maker to improve their predictions regarding an uncontrollable event such as market demand (e.g. Mock, 1971). Effective resource management requires somehow measuring the costs and benefits associated with resource use. While the costs generally can be ascertained, measuring their economic worth is frequently impossible. To enhance production, these firms can attempt to acquire more timely and accurate information regarding remanufacturing yields, or alternatively can attempt, to reduce the lead times of purchased parts (e.g. Ferrer & Ketzenberg, 2004).

Two main approaches have been used to incorporate information flow into inventory control and supply chains (e.g. Gavirneni, Kapuscinski, & Tayur, 1999). The first approach involves using the history of the demand process to more accurately forecast the demand distribution using Bayesian updates (e.g. Azoury, 1985; Lovejoy, 1990). Meanwhile, the second approach involves developing new analytical models (e.g. Chen, 1998; Gavirneni et al., 1999; Hariharan & Zipkin, 1995; Zheng & Zipkin, 1990; Zipkin, 1995). Therefore, this study is developing new analytical model and takes the point of view of the retailer and incorporates information flow between a retailer and customers.

There has been considered in the literature about the holding and penalty cost function. In some of the inventory models (e.g. Chang & Lin, 1991a; Chang, Lin, & Shen, 1996; Dekker, Frenk, Kleijn, & de Kok, 2000; Hollier, Mak, & Lam, 1995a, Hollier, Mak, & Lam, 1995b; Mak & Lai, 1995a, 1995b; Wu & Tsai, 2001; Wu, Lee, & Tsai, 2002), the holding and penalty cost functions are viewed as linear functions, that is, the holding and penalty rate are constant. In the other models (e.g. Chang & Lin, 1991b; Chen & Lin, 1989; Lin & Hwang, 1998; Lin & Tsai, 2003), the holding cost functions are viewed as concave functions with the properties $h(x) = 0$ for all $x \leqslant 0$ and $dh(x)/dx \geqslant 0$ and $d^2h(x)/dx^2 \leqslant 0$ for all $x \geqslant 0$. The penalty cost functions are viewed as concave functions with the properties $p(x) = 0$ for all $x \leqslant 0$, and $dp(x)/dx \geqslant 0$ and $d^2p(x)/dx^2 \leqslant 0$ for all $x \geqslant 0$. In this paper, the holding and penalty cost functions are viewed as Gauss functions which are applied such as the cost for mailing letters or packages in the post office and for shipping goods in containers.

In practice, many inventory systems must address sporadic demand patterns that may disrupt the inventory system. Customer

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doi:10.1016/j.eswa.2010.05.046

Nomenclature

N (Poisson distributed) number of customers arriving during a period;
λ arrival rate of customers, $\lambda = E(N)$;
Y_i order size of ith customer for $i = 1,2,\ldots,N$, random variable;
$a(j)$ order size distribution, i.e. $a(j) = P(Y_i = j)$; $a(0) = 0$;
M maximum order size, i.e. $M = \max\{j \geqslant 0 | a(j) > 0\}$;
q cutoff transaction size, decision variable;
I_A indicator function of the event A;
$a_q(j)$ order size distribution for orders smaller than the cutoff transaction size q, $a_q(j) = P(Y_i I_{\{0 \leqslant Y_i \leqslant q\}} = j)$;
D_q (compound Poisson distributed) demand during a period for orders smaller than the cutoff transaction size q, random variable, $D_q = \sum_{i=1}^{N} Y_i I_{\{0 \leqslant Y_i \leqslant q\}}$;
$f_q(j)$ distribution of demand during a period for orders smaller than the cutoff transaction size q, $f_q(j) = P(D_q = j)$;
$F_q(j)$ cumulative distribution of demand during a period for orders smaller than the cutoff transaction size q, $F_q(j) = P(D_q \leqslant j)$;
S order-up-to level, decision variable;
$S(q)$ optimal order-up-to level for cutoff transaction size q;
c unit ordering cost;
P_{rice} unit sale price;
$[x]$ n, $n \leqslant x < n+1$ for $n \in Z$;
$h(j)$ $-\left[-\frac{j}{k_1}\right]h$ = Gauss holding cost function which starts at $h(0) = 0$ and increases jumped a cost h for each k_1 units for units in stock at the end of the period with the properties $h(j) = 0$ for all $j \leqslant 0$, k_1, $h > 0$;
$p(j)$ $-\left[-\frac{j}{k_2}\right]p$ = Gauss penalty cost function which starts at $p(0) = 0$ and increases jumped a cost p for each k_2 units for unsatisfied demand during the period with the properties $p(j) = 0$ for all $j \leqslant 0$, k_2, $p > 0$;
$\pi(j)$ the overflow cost of alternatively satisfying an order size j for $j = q+1, q+2, \ldots, M$, $\pi(j) = \pi_0 + [c + \alpha(p(j) - c)]j$, π_0 is constant, $0 \leqslant \alpha \leqslant 1$.

orders exceeding the cutoff transaction size q units are treated as special orders to be satisfied by special direct deliveries from the factory to the customers to avoid disrupting the inventory system. Customer orders less than or equal to q units are met from stock. Subsequent papers, the inventory models of Dekker et al. (2000), Hollier et al. (1995a, 1995b), and Mak and Lai (1995a, 1995b) etc. considered a cutoff transaction size. Those authors offered an algorithm to determine the optimal values of reorder level s and order-up-to level S for a given q. This algorithm practically replaces the original order size distribution with a cutoff distribution, and used the algorithm of Zheng and Federgruen (1991) to obtain the optimal inventory policy.

According to the completeness of information, this work considers two models:

(1) Model 1: The retailer has incomplete information about the state of customers demand, namely the expected total profit of Dekker et al. (2000) is extended.
(2) Model 2: The retailer has full information regarding the state of customers demand.

Precise expressions are derived for the expected total profit of these two newsboy models with a cutoff transaction size and compound Poisson demand distribution (Adelson, 1966). In addition, this work establishes a new algorithm based on the Golden Section Search Technique (e.g. Haftka, Gurdal, & Kamat, 1990) for considering Gauss function to obtain the optimal order-up-to level S and the cutoff transaction size q. In the constantly changeful environment, it is worth measuring the value of information for enterprise decision making and whether enterprises will pay for information that can help them to increase profits. Thus, while information is always beneficial, this work attempted to investigate.

2. Model formulation

2.1. Model 1: uncertain demand model

This section extends the traditional newsboy model by incorporating a cutoff transaction size and the Gauss holding and penalty cost functions. In this new model, we address a variant of the holding and penalty cost function by considering the Gauss functions to fit in with some practical situations. Such situations occur commonly for mailing letters or packages in the post office or taking a taxi. In addition, a customer's order is only satisfied from stock if it does not exceed a prespecified cutoff transaction size. Otherwise, the customer is served in an alternative way, against additional cost.

Determining how the order size S and the cutoff transaction size q maximizes the expected total profit over a single period. Consequently, the expected total profit for a period is given by:

$$\begin{aligned} E(TP^u(S,q)) &= [\text{revenue} - (\text{ordering cost} + \text{holding cost} \\ &\quad + \text{penalty cost})] - (\text{overflow cost}) \\ &= E\left(TP_1^u(S,q)\right) - OC(q), \end{aligned}$$

where

$$\begin{aligned} E\left(TP_1^u(S,q)\right) &= \sum_{j=0}^{S} P_{rice} j f_q(j) + \sum_{j=S+1}^{\infty} P_{rice} S f_q(j) \\ &\quad - \left[cS + \sum_{j=0}^{S} h(S-j) f_q(j) + \sum_{j=S+1}^{\infty} p(j-S) f_q(j)\right], \end{aligned} \tag{1}$$

$$OC(q) = \lambda E\left(\pi(Y_i) I_{\{Y_i > q\}}\right) = \lambda \sum_{j=q+1}^{M} \pi(j) a(j),$$

$$h(j) = \begin{cases} -\left[-\frac{j}{k_1}\right]h, & 0 < j \leqslant S, \\ 0, & \text{otherwise}, \end{cases}$$

$$p(j) = \begin{cases} -\left[-\frac{j}{k_2}\right]p, & 0 < j, \\ 0, & \text{otherwise}, \end{cases}$$

$$a_q(j) = \begin{cases} \sum\limits_{i=q+1}^{M} a(i), & \text{if } j = 0, \\ a(j), & \text{if } j = 1,\ldots,q, \\ 0, & \text{otherwise}, \end{cases}$$

and

$$f_q(j) = \begin{cases} \exp(-\lambda(1 - a_q(j))), & \text{if } j = 0, \\ (\lambda/j) \sum\limits_{i=0}^{j-1} (j-i) a_q(j-i) f_q(i), & \text{if } j = 1,2,\ldots. \end{cases}$$

Our objective is to maximize the expected total profit over a single period; in other words, it is to find the solution of the optimization problem: