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Subjective modelling of supply and demand—the minimum of Fisher information solution

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ABSTRACT

Two of the present authors have put forward a projective geometry based model of rational trading that implies a model for subjective demand/supply profiles if one considers closing of a position as a random process. We would like to present the analysis of a subjectivity in such trading models. In our model, the trader gets the maximal profit intensity when the probability of transaction is ∼0.5853. We also present a comparison with the model based on the Maximum of Entropy Principle. To the best of our knowledge, this is one of the first analyses that show a concrete situation in which trader profit optimal value is in the class of price-negotiating algorithms (strategies) resulting in non-monotonic demand (supply) curves of the Rest of the World (a collective opponent). Our model suggests that there might be a new class of rational trader strategies that (almost) neglects the supply–demand profile of the market. This class emerges when one tries to minimize the information that strategies reveal.

1. Introduction

Conscious and rational economic activity requires, among others, an optimization of profit in given economic conditions and, usually, during more or less definite (basic) intervals. Usually, these intervals are chosen so that they contain a certain characteristic economic cycle (e.g. one year, an insurance period or a contract date). Often, it is possible (and always risky!) to make a prognosis for a more or less distant future of an undertaking by extrapolation from the already known facts, e.g. by various sorts of "statistical analyses". The quantitative description of an undertaking is extremely difficult when the duration of the intervals in question is itself a random variable (denoted by τ in the following). The profit gained during the specific period, represented as a function of τ, also becomes a random variable and that hardly can be used as an acceptable measure of the quality of the undertaking. To investigate activities that might have different periods of duration we define, following queuing theory [1], the *profit intensity* as a measure of this economic category [2]. A model of this type, although simple and elegant, has several drawbacks from the theoretical point of view. Besides, such phenomena perceived as games do not have any natural "quantum-like" version.[1] Such a possibility would be welcome because, for example, the appearance of non-Gaussian probability distribution functions suggest the existence of the so called Giffen goods [11]. Obstacles in constructing quantum-like versions of such models can be overcome by replacing the maximum Boltzmann/Shannon entropy principle with the requirement that the Fisher information gets its minimum under certain additional conditions (a discussion of the

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[1] "Quantization" often suggests ways of avoiding paradoxes in game theory due to the absence of limitations of the classical theory of probability. This approach has interesting consequences in decision sciences, cf. for example papers by Aerts and Czachor [3], Haven [4,5], Khrennikov [6–8], Piotrowski and Sładkowski [9], Yukalov and Sornette [10] and others. Of course, we do not claim that quantum processes play any explicit role here.

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connection between the principle of the minimum of Fisher information and equations of quantum theory can be found in Ref. [12]). In this way, a simple method of quantum-like reformulation in terms of game theory that stem from statistical considerations is possible. Moreover, this approach allows for analysis of subjectivity in strategy selection. This paper is in some sense an extension and continuation of our previous work [13] and is organized as follows. First we describe the merchandizing mathematician model (MM) put forward in Ref. [2] and quote the relevant definitions. Then we argue for the use of logarithmic quotations and define the logarithmic rate of return and briefly describe its scaling invariance [14]. The results showing the usefulness of Fisher information as a criterion for strategy selection and estimation of probabilities of making profits will be given in Section 3. Finally we will point out some issues that still need to be addressed.

2. The merchandizing mathematician model

The merchandizing mathematician model [2] describes a repeated buying–selling process on a liquid market with minimal assumptions about the market and the decision process. This model, although very simple, has interesting recommendations for stock exchange traders: *do not try to beat yourself and accept trading profits that are equal to or better than your average record*. Below we describe its main features that will be used in our analysis.

2.1. The profit intensity

To proceed, let us denote by t, υ_t and $\upsilon_{t+\tau}$ the beginning of an interval of the duration τ, the value of the undertaking (asset) at the beginning and at the end of the interval in question, respectively. We define the *logarithmic rate of return* $r_{t,t+\tau}$ as

$$r_{t,t+\tau} \equiv \ln\left(\frac{\upsilon_{t+\tau}}{\upsilon_t}\right). \tag{1}$$

Let the expectation value of the random variable ξ in one cycle (buying–selling or vice versa) be denoted by $E(\xi)$. Then, if $E\left(r_{t,t+\tau}\right)$ and $E(\tau)$ are finite, we can define the *profit intensity* for one cycle ρ_t as

$$\rho_t \equiv \frac{E\left(r_{t,t+\tau}\right)}{E(\tau)}. \tag{2}$$

This definition is an immediate consequence of the Wald identity [15]:

$$E\left(S_{\tau'}\right) = E\left(X_1\right) E\left(\tau'\right), \tag{3}$$

where $S_{\tau'} \equiv X_1 + \cdots + X_{\tau'}$ is the sum of τ' equally distributed random variables $X_k,\ k = 1, \ldots, \tau'$ and τ' is the stopping time [1,15]. The profit intensity Eq. (2) is just the expectation value of X_1 in the Wald identity Eq. (3). The expected profit is the left hand side of the Wald identity. If we are interested in the profit expected in a time unit, we have to divide, according to the Wald identity Eq. (3), the expected profit by the expectation value of the stopping time. Therefore we obtain the formula given by Eq. (2). We can also calculate the variance of the profit intensity by using Proposition 10.14.4 from Resnick's book [15]:

$$E\left(\left(S_{\tau'} - \tau' E\left(X_1\right)\right)^2\right) = E\left(\tau'\right) \operatorname{Var}\left(X_1\right). \tag{4}$$

2.2. The merchandizing mathematician model

The simplest market consists in exchanging two goods which we would arbitrarily call *the asset* and *the money* and denote them by Θ and \$, respectively. The model consists in the repetition of two simple basic moves (in principle, the process is continued endlessly):

1. First move consists in a rational buying of the asset Θ (exchanging \$ for Θ).
2. The second move consists in a selling of the purchased amount of the asset Θ (exchanging Θ for \$).

The rational buying is simply a purchase bound by a fixed *withdrawal price* $-a$ that is a logarithmic quotation for the asset Θ, $-a$, above which the trader gives the buying up. A random selling can be identified with the situation when the withdrawal price is set to $-\infty$ (the trader in question is always bidding against the rest of the traders). The order of these transactions can be reversed and, in fact, is conventional. Note that the quotation method does not matter to the discussed process as long as it can be repeated many times. Let V_Θ and $V_\$$ denote some given amounts of the asset and the money, respectively. If at some time t the assets are exchanged in the proportion $V_\$: V_\Theta$ then we call the number

$$p_t \equiv \ln\left(V_\$\right) - \ln\left(V_\Theta\right) \tag{5}$$

the logarithmic quotation for the asset Θ. If the trader buys some amount of the asset Θ at the quotation p_{t_1} at the moment t_1 and sells it at the quotation p_{t_2} at the later moment t_2 then his profit (or more precisely the logarithmic rate of return)