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A multi-grid framework for the extraction of large-scale vortices in Large-Eddy Simulation

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ABSTRACT

The analysis of large-scale vortices from highly refined unsteady simulations becomes challenging as the mesh resolution increases. Beyond the large amount of data that needs to be processed, classical vortex visualization techniques based on invariants of the velocity gradient tensor fail in extracting the large-scale vortices as the velocity gradient tensor magnitude is greater for small turbulent eddies than for energy-containing vortices. This problem is even more important in highly-resolved simulations with a broad range of eddies. The methodology presented here is a geometric multi-grid high-order filtering (MGHOF) framework for on-line analysis of high-fidelity simulations. This approach relies on high-order implicit filters and enables the extraction of large-scale features from Large-Eddy Simulations (LES) on massive and distributed unstructured grids at a reduced cost. The MGHOF framework is first described and validated, then the methodology is applied to a 3D turbulent plane jet and to the LES of a 3D low-Mach number turbine blade with various mesh sizes, ranging from a few million to a few billion tetrahedra. In the latter case, the MGHOF enables to perform the dynamic mode decomposition of the velocity and temperature fields for the finer grid resolution.

1. Introduction

LES has proven being a valuable tool to study 3D turbulent flows in complex geometries as those found in automotive engines or aeronautical gas turbines. In such flows, LES enables to analyze the dynamics of coherent structures [1], which play an important role in the device performances and operability. Moreover, the steady increase in computational power leads to finer mesh resolutions, which allow to access more physical phenomena, but also increase the difficulty to analyze the large scale dynamics.

The vortex identification techniques based on the velocity gradient tensor invariants are precious tools to visualize the coherent flow features. Those methods allow to understand and qualify the turbulent structures as they identify fluid regions where eddies with a high rotational rate are present. Several vortex identifiers have been introduced in the literature: Hunt, Wray, and Moin [2] proposed the well known Q-criterion, which is the second invariant of the deformation tensor, and can be expressed as

$$Q = \frac{1}{2}(||\boldsymbol{\Omega}||_F^2 - ||\boldsymbol{S}||_F^2)\,, \tag{1}$$

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where $||.||_F$ refers to the Frobenius norm and

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right), \tag{2}$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{3}$$

Positive values of the Q-criterion are found at the core of vortices as the rotation rate $\boldsymbol{\Omega}$ is higher than the strain rate $\boldsymbol{S}$ [3,4]. Another important feature of the Q-criterion is to have zero values at the wall as the two components, rotation and strain rate, have the same magnitude. Similarly, Jeong and Hussain [4] proposed another criterion based on the idea that a pressure minimum corresponds to the presence of a vortex core. Neglecting the terms due to vorticity transport, irrotational strain and viscous effects, the equation of the Hessian of pressure is formed by taking the gradient of the Navier–Stokes equation as

$$-\frac{1}{\rho}(\nabla \otimes \nabla P) = \boldsymbol{\Omega}^2 + \boldsymbol{S}^2. \tag{4}$$

It can be shown from Eq. (4) that a pressure minimum appears when two of the three real eigenvalues of the tensor $\boldsymbol{\Omega}^2 + \boldsymbol{S}^2$ are negative. Then the classical λ_2 vortex identifier, where λ_2 is the intermediate eigenvalue, can be introduced when ordering the eigenvalues as $\lambda_1 \geq \lambda_2 \geq \lambda_3$. The vortex identification criterion is defined as $\lambda_2 < 0$ corresponding to the presence of a vortex core.

The difference between those two vortex identifiers comes from the fact that the Q-criterion deals with the excess of rotation over the strain rate in all spatial direction whereas the λ_2 criterion only considers it in a specific plane [5]. As highlighted by several studies [3,6], iso-surfaces of the Q-criterion and the λ_2 criterion give very similar results when equivalent threshold are used. Chakraborty et al. [5] also demonstrated that for an inviscid incompressible flow, the classical relation between the pressure Laplacian, the fluid density ρ and the Q-criterion is found by taking the trace of Eq. (4): $\Delta P = 2\rho Q$. It highlights again the relation between both criteria.

While the Q-criterion successfully extracts the largest vortices in laminar flows or in LES with coarse meshes, its fails at identifying the large scale dynamics in very refined simulations. This issue comes from the scaling of the Q-criterion, which is larger for small vortices than for energy carrying eddies in an isotropic homogeneous turbulence. This can be shown using a simple vortex theory such as the Lamb Oseen vortex. In this case, the velocity in the circumferential direction θ is defined as $u_\theta(r,t) = \Gamma/(2\pi r)\left(1 - \exp(-r^2/r_c^2)\right)$ where Γ is the circulation and r_c the radius of the vortex core. The maximum of the Q-criterion is found at the center of the vortex as the shear component vanishes with $Q_{max} \sim \Gamma^2/r_c^4$. The enstrophy $\mathcal{E}$, i.e. the integral of the squared vorticity, for this vortex is equal to $\Gamma^2/(2\pi r_c^2)$. In the inertial range of homogeneous isotropic turbulence, enstrophy scales as $k^{1/3}$, which implies that the Q-criterion maximum scales as $k^{7/3}$. With this scaling, small vortices have higher values of Q-criterion than large scales. Then, in highly-resolved LES, the small vortices might completely mask large vortices when plotting Q-criterion iso-surfaces.

To circumvent the Q-criterion scaling issue, it is mandatory to develop numerical techniques capable of separating the different coherent structures such as spatial low-pass filters. Performing this scale separation is quite challenging as it requires to extract features from a large amount of data distributed across a large number of processors in a parallel environment. Moreover, such a low-pass filter necessitates a good selectivity in order to leave the large scales unaffected while damping all the smallest scales. Performing the scale separation in complex geometries with unstructured grids is also challenging as the stencil of differential operators is generally limited to the closest neighboring control volumes. Finally, in these geometries, the filter kernel degeneracy at the boundaries is also an important issue.

The present geometric multi-grid high-order filtering (MGHOF) framework has been designed to circumvent all these issues and to enable an efficient extraction of large scale features in turbulent flows. It relies on a hierarchy of grids, where the highly-resolved LES data are successively filtered and interpolated on coarser grids. On these coarser grids, the data volume is dramatically reduced. The application of modal decomposition methods such as Proper Orthogonal Decomposition (POD) [7–9], or Dynamic Mode Decomposition (DMD) [10,11] becomes tractable on the coarsest grids, as these methods require the storage of large amount of snapshots, which is presently intractable for billion-cell simulations without sub-sampling.

All the numerical simulations presented in the article have been performed using the finite-volume code YALES2 [12], a Large-Eddy Simulation and Direct Numerical Simulation (DNS) solver based on unstructured meshes. This code solves the low-Mach number Navier–Stokes equations for turbulent reactive flows using a time-staggered projection method for constant [13] or variable density flows [14]. YALES2 is specifically tailored to solve these low-Mach number equations on massively parallel machines on billion-cell meshes thanks to highly optimized linear solvers [15].

This paper is organized in five parts. First, the high-order implicit filters are presented and analyzed in terms of CPU cost. The MGHOF framework is then exposed. The validation and the choice of the main parameters of the MGHOF are discussed in a third part. Then, the performances of the MGHOF framework are assessed for a 3D turbulent jet plane, where the large-scale dynamics is exhibited. Finally, this framework is applied to the LES of a low-Mach number turbine blade with meshes up to 2.2 billion cells to highlight its ability to extract the large-scale vortices on-line at a limited CPU cost. Dynamic mode decomposition complements the analysis of the flow dynamics.