The spectral nodal method applied to multigroup $S_N$ neutron transport problems in One-Dimensional geometry with Fixed–Source

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ABSTRACT

The nodal methods, as deterministic models, form a class of numerical methods developed to generate accurate numerical solutions of the Boltzmann equation for neutron transport. These methods are algebraically and computationally more laborious than the traditional deterministic fine-mesh methods like the Diamond Difference method (DD). However, their numerical solution for traditional coarse-mesh problems is more accurate. For this reason, the nodal methods and their algorithms for direct and iterative solution schemes have been the subject of extensive research. In this paper we propose a simpler methodology for the development of a method of spectral nodal class which is tested as an initial study of the solutions (spectral analysis) of the neutron transport equation in the formulation of discrete ordinates ($S_N$), in one-dimensional geometry, multigroup energy approximation, isotropic scattering and considering homogeneous and heterogeneous domains. These results are compared with the traditional fine-mesh DD method and the spectral nodal methods, spectral Green’s function (SGF) and Response Matrix (RM) to test their numerical accuracy, stability and consistency.

1. Introduction

Neutrons play a significant role in many nuclear reactions. They are responsible for propagating the chain reaction in a nuclear reactor and their radiation can be penetrating and ionizing, so it can either be used for medical treatments or to determine the amount of water present in the soil.

For developing the design and analysis of nuclear reactors, accurate and detailed prediction of the neutron distribution in space and time, as well as its energy dependence in all components of the reactor, is needed. This distribution is obtained by solving the linearized neutron transport equation, which is an adaptation of the Boltzmann equation, first formulated for the study of the kinetic theory of gases (Duderstadt and Martin, 1979), (Lewis and Miller, 1993). This linearized transport equation is an integro-differential equation that has seven independent variables to describe the average behavior of the entire neutron population. Its analytical solution is complicated except for highly idealized problems. Its exact solution can only be obtained for the less complex problems; for more complex problems, approximate solutions can be obtained applying numerical techniques. Deterministic numerical methods have been developed for obtaining approximate but accurate solutions to the radiation shielding problems, global reactor calculations and other applications.

These numerical methods make possible the computer modeling of nuclear systems and radiation shield problems using a deterministic approach since they usually use the formulation of discrete ordinates ($S_N$) (Lewis and Miller, 1993) and the multigroup approximation to discretize the angular and energy variables in the transport equation. The multigroup approximation divides the particle energy range into contiguous energy groups and the $S_N$ discrete ordinates formulation in slab geometry problems, made a collocation scheme for the angular variables in prescribed directions (discrete ordinates) and use an angular quadrature sets for the approximation of the integral source terms.

Based mainly on these approximations different methods for solving the linearized neutron transport equation were developed, such as the fine-mesh Diamond Difference method (DD) (Duderstadt and Martin, 1979), the coarse-mesh methods spectral Green’s function (SGF) (Barros, 1990) and Response Matrix (RM) (Silva et al., 2013). The DD method is based on the linear approximation of the neutron angular flux, where the value of the average angular flux at each cell is the arithmetic mean of the angular fluxes at the spatial cell interfaces (Lewis and Miller, 1993). The SGF method was introduced by R.C. Barros & E.W. Larsen (Barros and Larsen, 1990) in 1990; they presented...
this approach for a general one-group, slab geometry, discrete ordinates problems with linearly anisotropic scattering and a prescribed internal source, which produced solutions that were completely free of spatial truncation errors (Barros and Larsen, 1990).

In this paper, we describe and test a new numerical nodal methodology framed in the coarse-mesh methods, aimed to develop a method of spectral nodal class (Barros, 1990). This method is an initial study of the solutions (spectral analysis) of the multigroup neutron transport equation in slab geometry, in the formulation of discrete ordinates (SD), considering isotropic scattering in homogeneous and heterogeneous domains with a prescribed internal source. This strategy is called Spectral Deterministic Method (SDM), and like the SGF and RM methods, it is free of spatial truncation errors in one-dimensional problems in one group and multigroup energy formulations.

The SDM uses the estimates of the incident angular fluxes in a cell to determine the outgoing angular fluxes in all directions, initially obtained by the general solution of the transport equation in discrete ordinates formulation. From this solution we get the particular and the homogeneous components. The homogeneous component leads to an eigenvalue problem from which the eigenvalues and eigenvectors are obtained for each cell, and the particular component is a constant function which depends on the energy groups of neutron source and its position in the problems analyzed. With the calculated eigenvalues, eigenvectors, particular solution and the estimated incident fluxes, we can calculate the outgoing angular fluxes at cell-edge. The SDM uses the most recent estimates of the incoming angular fluxes in the cell interface for all energy groups, to calculate the outgoing angular fluxes in the discrete ordinates directions, which become the incoming fluxes to the adjacent cell in the discrete ordinates directions of the transport sweep.

This paper is organized as follows: Section 2 presents the derivation of the spectral analysis of multigroup transport equations in discrete ordinates formulation. In Section 3, the iterative methodology of the new multigroup Spectral Deterministic Method (SDM) is introduced and described. Section 4 presents the comparison of the numerical results of the SDM with the results of the DD, SGF and RM methods to verify its accuracy and consistency. Finally, Section 5 contains the discussion of the results and suggestions for future work.

2. Spectral analysis of the multigroup slab geometry

The equation that mathematically models the multigroup neutron transport in slab geometry, in a system with isotropic scattering, time-independent and fixed-source ($Q_k$), is represented by

$$\mu \frac{\partial \psi(x, \mu)}{\partial x} + \sigma_f(x) \psi(x, \mu) = \sum_{g=1}^{G} \frac{\sigma_{0 \rightarrow g}(x)}{2} \int_{-1}^{1} \psi(x, \mu') d\mu' + Q_k(x), \quad g = 1:G. \tag{1}$$

Considering the multigroup $S_0$ formulation in a slab $\Gamma$, Eq. (1) takes the form

$$\mu_{m} \frac{d}{dx} \psi_{m,g}(x) + \sigma_{0}^{\gamma}(x) \psi_{m,g}(x) = \sum_{g=1}^{G} \frac{\sigma_{0 \rightarrow g}(x)}{2} \sum_{n=1}^{N} \omega_{n} \psi_{m,n}(x) + Q_{f}(x), \quad m = 1: N, \quad g = 1:G. \tag{2}$$

Here $x \in \Gamma, 1 \leq g \leq G$, where $G$ represents the number of energy groups, and $1 \leq m \leq N$, where $N$ is the order of the Gauss-Legendre quadrature set (Lewis and Miller, 1993), which is the quadrature set used for the resolution of the $S_0$ problem in this paper. For each region analyzed, $\sigma_{0}^{\gamma}(x)$ describe the $g$-th group macroscopic total cross section, which includes all possible interactions. $\sigma_{0 \rightarrow g}(x)$ is the macroscopic $g$-th isotropic differential scattering cross section from group $g'$ to group $g$, and $Q_f(x)$ is the isotropic interior source in energy group $g$. The angular flux of the particles traveling in the discrete ordinates direction $\mu_{m}$ for each group is represented by $\psi_{m,g}(x)$ and $\omega_{n}$ is the quadrature weight.

Let us consider an arbitrary $\Gamma$ spatial grid on the domain $D$, as shown in Fig. 1, where each spatial cell $\Gamma_{j}$ have width $h_{j}$ and constant cross sections $\sigma_{0 \rightarrow g}$ and $\sigma_{0}^{\gamma}$. The equation who describes equation (2) in the formulation of $\sum_{g}$ discrete ordinates, defined in an arbitrary homogeneous cell has the form

$$\mu_{m} \frac{d}{dx} \psi_{m,g}(x) + \sigma_{0}^{\gamma}(x) \psi_{m,g}(x) = \sum_{g=1}^{G} \frac{\sigma_{0 \rightarrow g}(x)}{2} \sum_{n=1}^{N} \omega_{n} \psi_{m,n}(x) + Q_{f}(x), \quad x \in \Gamma_{j}, \quad m = 1: N, \quad g = 1:G. \tag{3}$$

The general solution of the system of equation (3) is given by

$$\psi_{m,g}(x) = \psi_{m,g}^{\in}(x) + \psi_{m,g}^{\out}(x), \tag{4}$$

where the superscript $p$ denotes the particular solution with fixed-source and $h$ indicates the homogeneous component of the local general solution, which satisfies the system of equation (3). The particular solution $\psi_{m,g}^{\out}$, with fixed-source ($Q_{f}$), takes the form

$$\sum_{g=1}^{G} \left( \sigma_{0}^{\gamma} \delta_{k-\epsilon} - \sigma_{0 \rightarrow g}(x) \right) \psi_{m,g}^{\out} = Q_{f}, \quad g = 1:G. \tag{5}$$

Here, we have defined $\delta_{k-\epsilon}$ as the Kronecker delta. To determine the homogeneous solution $\psi_{m,g}^{h}$ associated with system (2), we first set $Q_{f} = 0$, and then seek for elementary solutions of the form

$$\psi_{m,g}(x) = a_{m,g} \left( \frac{x - x_{\Gamma_{j}} - \frac{1}{2}}{h_{j}} \right), \quad m = 1: N, \quad g = 1:G, \quad x \in \Gamma_{j}, \tag{6}$$

where $x_{\Gamma_{j}} - \frac{1}{2}$ represent the $\Gamma_{j}$ cell left boundary. Inserting Eq. (6) on the Eq. (3), we obtain the eigenvalue problem

![Fig. 1. Spatial cell $\Gamma_{j}$ in a one-dimensional domain $D$ with length $H$.](image1)

![Fig. 2. Homogeneous slab of thickness $H$.](image2)

![Fig. 3. Homogeneous slab of thickness $H$.](image3)
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