Optimal investment strategies for an insurer and a reinsurer with a jump diffusion risk process under the CEV model

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ABSTRACT

In this paper, we consider the optimal investment problem for both an insurer and a reinsurer. The insurer's wealth process is described by a jump diffusion risk model and the insurer can purchase proportional reinsurance from the reinsurer. Both the insurer and the reinsurer are allowed to invest in a risk-free asset and a risky asset whose price process follows the constant elasticity of variance (CEV) model. Moreover, the correlation between risk model and the risky asset's price is considered. The objective is maximizing the expected utility of the insurer's and the reinsurer's terminal wealth. Applying stochastic control theory, we establish the corresponding Hamilton–Jacobi–Bellman (HJB) equations and derive optimal investment–reinsurance strategies for exponential utility function. Finally, numerical examples are provided to analyze the effects of parameters on the optimal strategies.

1. Introduction


In the above-mentioned literatures, they generally assume that the risky asset's prices are driven by geometric Brownian motions (GBMs), which implies the volatilities of the risky asset's prices are constant and deterministic. But empirical...
analysis has shown that the volatility is not constant, see [13], and the references therein. In this paper, we assume that the price process of the risky asset follows the constant elasticity of variance (CEV) model, which is a natural extension of geometric Brownian motion (GBM) and more practical. Moreover, the CEV model can explain the implied volatility skew and it is analytically tractable in comparison with other stochastic volatility models. The CEV model was proposed by Cox and Ross [14]. Beccers [15], Davydov and Linetsky [16], Jones [17] studied the option pricing problems under the CEV model. Xiao et al. [18] applied the CEV model to investigate the investment problem for pension plan and derived the optimal strategy for lognormal utility function by using Legendre transform and dual theory. Gao [19,20] studied the investment problem for pension plan and obtained the optimal solutions for CRRA and CARA utility functions. Nowadays, the CEV model has also been commonly used in optimal reinsurance and investment problems. Gu et al. [21] considered optimal investment and proportional reinsurance problem for utility maximization. Liang et al. [22] and Lin and Li [23] assumed the price of risky asset followed the CEV model and studied the proportional reinsurance problem under the jump diffusion risk model.

However, most of the researches mentioned previously only consider the optimal strategy for an insurer. But in practice, the optimal reinsurance strategy for an insurer may not be optimal for a reinsurer. Thus, it is necessary to take the management of the reinsurer into account. Currently, some researchers began to study optimal investment–reinsurance problem for both the insurer and the reinsurer. For example, Li et al. [24,25] studied the optimal investment problem for an insurer and a reinsurer for utility maximization. Zhao et al. [26], Li et al. [27] investigated the time-consistent reinsurance–investment strategy for an insurer and a reinsurer under a mean–variance framework. But most of them describe the basic risk process by a Brownian motion with drift. In this paper, we describe the insurer’s wealth process by jump diffusion risk model, which is a compound Poisson process perturbed by a Brownian motion. Both the insurer and the reinsurer are allowed to invest in a risk-free asset and a risky asset whose price process follows the CEV model. Moreover, we consider the correlation between risk model and the risky asset’s price. The objective is to maximize the expected utility of the insurer’s and the reinsurer’s terminal wealth. By solving the corresponding Hamilton–Jacobi–Bellman (HJB) equations via Legendre transform and dual theory, closed-form solutions to the problems of expected exponential utility maximization are derived under some given assumptions. Furthermore, numerical examples are presented to analyze the effects of parameters on the optimal strategies.

This paper is organized as follows. In Section 2, we introduce the formulation of the model. Section 3 and Section 4 derive the optimal investment–reinsurance strategies to maximize the utility of the insurer’s and reinsurer’s terminal wealth. In Section 5, numerical examples are carried out to analyze the effects of parameters on the optimal strategies. Finally, we give conclusions in Section 6.

2. Model formulation

In this paper, we consider a filtered complete probability space \((\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \in [0,T]}, P)\) satisfying the usual condition, where \(\mathcal{F}_t\) is the information of the market available up to time \(t\), \([0,T]\) is a fixed and finite time horizon. All stochastic processes introduced below are assumed to be well-defined and adapted to \(\{\mathcal{F}_t\}_{t \in [0,T]}\).

2.1. Wealth process of the insurer

Without reinsurance and investment, the wealth process of the insurer is described by the jump diffusion risk model

\[
dX(t) = cdt - d\left(\sum_{i=1}^{N(t)} Z_i\right) + \beta d\bar{W}(t),
\]

where \(c\) is premium rate of the insurer, \(\sum_{i=1}^{N(t)} Z_i\) is a compound Poisson process representing the cumulative amount of claims in time interval \([0, t]\). \(\{N(t), t \geq 0\}\) is a homogeneous Poisson process with intensity \(\lambda > 0\) and the claim sizes \(\{Z_i(i \geq 1)\}\) are independent and identically distributed (i.i.d.) positive random variables with common distribution \(F(z)\) and independent of \(N(t)\). Denote the mean value \(E[Z_i] = \mu_s\) and moment generating function \(M_2(r) = E[e^{rZ_i}]\). We assume that \(E[e^{rZ_i}] = M_2'(r)\) exists for \(0 < r < \zeta\) and that \(\lim_{r \to \zeta^-} E[e^{rZ_i}] = \infty\) for some \(0 < \zeta \leq +\infty\), \(\beta > 0\) is a constant, and \(\{\bar{W}(t)\}_{t \geq 0}\) is a standard Brownian motion. Suppose the premium is calculated according to the expected value principle, i.e., \(c = (1 + \eta)\lambda\mu_s\), where \(\eta > 0\) is the positive safety loading of the insurer. The diffusion term \(\beta d\bar{W}(t)\) represents the uncertainty related to the insurer’s wealth process at time \(t\).

The insurer is allowed to purchase proportional reinsurance from the reinsurer to hedge insurance risk. Let \(q_1(t) \in [0, 1]\) be the reinsurance proportion, that is, when the \(i\)th claim \(Z_i\) occurs, the insurer pays only \(q_1(t)Z_i\) while the reinsurer pays \((1 - q_1(t))Z_i\). The reinsurance premium is calculated according to the expected value principle, i.e., \(\delta(q_1) = (1 + \theta)(1 - q_1(t))\lambda\mu_s\), where \(\theta > \eta\) is the safety loading of the reinsurer. Moreover, the insurer is allowed to invest in a risk-free asset and a risky asset. The price process of the risk-free asset \(S_0(t)\) is given by

\[
dS_0(t) = rS_0(t)dt, \quad S_0(0) = s_0,
\]

where \(r > 0\) is the risk-free interest rate. The price process of the risky asset \(S(t)\) is described by the CEV model:

\[
dS(t) = rS(t)dt + \sigma(S(t))^{k + 1}dW(t), \quad S(0) = s.
\]
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