On dynamic investment strategies

John C. Cox\textsuperscript{a}, Hayne E. Leland\textsuperscript{b,*}

\textsuperscript{a}Sloan School, MIT, 50 Memorial Drive, Cambridge, MA 02139, USA
\textsuperscript{b}Haas School of Business, University of California at Berkeley, Berkeley, CA 94720, USA
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Abstract

This paper presents a new approach for analyzing dynamic investment strategies. Previous studies have obtained explicit results by restricting utility functions to a few specific forms; not surprisingly, the resultant dynamic strategies have exhibited a very limited range of behavior. In contrast, we examine what might be called the inverse problem: given any specific dynamic strategy, can we characterize the results of following it through time? More precisely, can we determine whether it is self-financing, yields path-independent returns, and is consistent with optimal behavior for some expected utility maximizing investor? We provide necessary and sufficient conditions for a dynamic strategy to satisfy each of these properties. \copyright\ 2000 Published by Elsevier Science B.V. All rights reserved.

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\textsuperscript{*}Corresponding author.

\textsuperscript{a}Editor's note: Although presented in the privately circulated Proceedings of the Seminar on the Analysis of Security Prices, Center for Research in Security Prices, University of Chicago, this paper has remained unpublished for a variety of reasons. For almost 20 years it has been frequently referred to and used by many financial economists, in areas as diverse as portfolio insurance and risk premium analysis. I am certain that its publication will ensure its wider availability and further research into fundamental issues.
1. Introduction

This paper presents a new approach for analyzing dynamic investment strategies. Previous studies have obtained explicit results by restricting utility functions to a few specific forms; not surprisingly, the resultant dynamic strategies have exhibited a very limited range of behavior.\(^1\) In contrast, we examine what might be called the inverse problem: given any specific dynamic strategy, can we characterize the results of following it through time? More precisely, can we determine whether it is self-financing, yields path-independent returns, and is consistent with optimal behavior for some expected utility maximizing investor? We provide necessary and sufficient conditions for a dynamic strategy to satisfy each of these properties.

Our results permit assessment of a wide range of commonly used dynamic investment strategies, including ‘rebalancing’, ‘constant equity exposure’, ‘portfolio insurance’, ‘stop loss’, and ‘dollar averaging’ policies.

Indeed, any dynamic strategy that specifies the amount of risky investment or cash held as a function of the level of investor wealth, or of the risky asset price, can be analyzed with our techniques.

We obtain explicit results for general dynamic strategies by assuming a specific description of uncertainty. We consider a world with one risky asset (a stock) and one safe asset (a bond). We assume that the bond price grows deterministically at a constant interest rate and that the stock price follows a multiplicative random walk that includes geometric Brownian motion as a limiting special case. This limiting case, which implies that the price of the risky asset has a lognormal distribution, has been widely used in financial economics.\(^2\) The restriction to a single risky asset involves no significant loss of generality, since it can be taken to be a mutual fund.\(^3\) Furthermore, our basic approach can be applied to other kinds of price movements. We assume that the risky asset pays no dividends. We explain later why this too involves no real loss of generality. We allow both borrowing and short sales with full use of the proceeds. We further assume that all markets are frictionless and competitive.

We wish to make full use of the tractability that continuous time provides for characterizing optimal policies. However, we appreciate the view that the economic content of a continuous-time model is clearer when it is obtained as the limit of a discrete-time model. Accordingly, we always establish our results in a setting in which trading takes place at discrete times and the stock price

\(^{1}\) For example, see Hakansson (1970), Levhari and Srinivasan (1969), Merton (1971), Phelps (1962), and Samuelson (1969).

\(^{2}\) For example, see Merton (1969) and Merton (1971). See also Black and Scholes (1973) and much of the extensive literature based on their article.

\(^{3}\) See Merton (1971) and Ross (1978).
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