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An approach based on Hotelling's test for multicriteria stochastic simulation–optimization

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Abstract

In a stochastic simulation context, iterative methods of optimization, which perform at each step of their optimization procedure a comparison between two different values of the objective function, need the use of statistical tests in order to properly evaluate and compare the simulation results. However, when the objective function to be optimized is a multicriteria function involving several performance measures, classical statistical procedures, which do not take into account the correlation between the performance measures, could reject acceptable solutions. To avoid this, we propose an efficient and rigorous statistical procedure already used in a multicriteria context, Hotelling's T^2 procedure. This paper shows that this procedure is very well adapted when the problem is to compare simultaneously several criteria in a stochastic simulation–optimization context.

Keywords: Stochastic simulation–optimization; Multivariate tests; Job shop

1. Introduction

A stochastic simulation model is a model with random input variables. Consequently, performance measures are also random variables, which may be evaluated through their means μ. However, each run of the simulation model gives only an estimate of the true value of μ. In order to evaluate the precision of the estimation, it is necessary to calculate a confident interval on μ.

Frequently, a simulation model is associated with measures of the system performance, $f(z)$, which has to be optimized. Since iterative methods only require, point by point, the evaluation of the objective function, and not derivatives, these methods

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are often used in simulation–optimization problems where the simulation model is complex and cannot be expressed analytically in terms of the model variables. Popular iterative methods are based:

- either on direct search methods [3,5,13,14] such as the Hooke and Jeeves [6] method, where a comparison is made – at each new evaluation of the objective function – between the current optimum (i.e., the best value obtained up to that iteration), and the challenger point (i.e., the new value of the objective function at that point);
- or on random optimization techniques such as simulated annealing, taboo search, or evolutionary strategies [4,10]. These optimization methods also require the comparison – at each step in their optimization procedure – between a reference value and a challenger value.

In stochastic simulation, the proper comparison of two solutions needs statistical tests such as a comparison of two-sample means [2,8]. This is appropriated when comparing one criterion on two samples to determine whether the two samples belong to the same population or not, i.e., the observed criterion has the same mean for the two samples, assuming that both samples have the same variance [2,8,12].

But, when the objective function is a multicriteria function, this breaks down. The aims of this paper are:

1. to highlight the theoretical problems caused by the simultaneous comparison of several criteria in a stochastic simulation–optimization environment;
2. to propose two solutions to these problems.

The first solution has already been discussed in [12], the second, more general, will be presented in this paper. We will compare this new solution with the classical procedures using an example based on a manufacturing system model.

2. Theoretical problems

2.1. Comparison of two-sample [2,8]

Let X be the random variable measuring the performance of the system, and let this random variable be evaluated through its mean μ. By using appropriate methods (e.g., independent replications, batch means) we may suppose that the simulation gives a set of n independent and identically distributed (i.i.d.) random variables $\{X_1, X_2, \ldots, X_n\}$. Then, $\overline{X} = (1/n)\sum_{i=1}^{n} X_i$ is an unbiased estimator of the true mean μ. If the number of observations n is large enough, $\overline{X}$ converges to a normal random variable with mean μ. Thus, the classical student test can be used to compare two strategies:

$$\begin{aligned} \mathrm{H}_0: &\quad \mu_j - \mu_{\mathrm{opt}} = 0, \\ \mathrm{H}_1: &\quad \mu_j - \mu_{\mathrm{opt}} \neq 0, \end{aligned}$$

with μ_j measuring the system response at the jth iteration (i.e., the challenger point), and μ_{opt} measuring the best system performance obtained up to that iteration (i.e., the current optimum).