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Optimal investment strategy for defined contribution pension schemes

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Abstract

We analyse the financial risk in a defined contribution pension scheme, applying dynamic programming techniques to find an optimal investment strategy for the scheme member. We use a series of interim targets and a target at retirement linked to the desired net replacement ratio. We consider both the investment risk and the annuitisation risk faced by the individual and specifically consider the properties of the so-called "lifestyle" investment strategies. The principal results concern the suitability of the lifestyle strategy and the large variability of the level of pension achieved at retirement in the case of a variable annuity conversion rate. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

The present work analyses the financial risk in defined contribution pension schemes and uses stochastic dynamic programming in an attempt to find an optimal investment strategy, given a final target linked to the net replacement ratio and a set of interim targets. [2]

The use of dynamic programming is not new in the actuarial context. Haberman and Sung (1994) use Bellman's optimality principle to minimise simultaneously the contribution risk and the solvency risk in a defined benefit pension scheme, and derive the optimal contribution rate. Cairns (1997), in a continuous time framework, and Owadally (1998), in a discrete time framework, apply this principle to a defined benefit scheme in order to derive the optimal contribution rate and the optimal allocation decision in a two-asset world.

Thomson (1998) has applied multi-period methods to defined contribution pension schemes, and has determined the optimal investment strategy maximising the member's expected utility.

In the analysis that follows, both the investment risk and the annuity risk borne by the member of the pension scheme are studied, where by "investment risk", we mean the risk that a poor investment performance during the active membership leads to a lower than expected accumulated fund, and by "annuity risk", we mean the risk that

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[1] http://www.city.ac.uk/actstat/dept/shwww.html.

[2] A fuller analysis and set of results can be found in Vigna (1999).

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a low conversion rate used in buying the annuity at retirement leads to a lower than expected pension rate.[3] The suitability of the lifestyle policy is also discussed. (This is clearly described by Sze (1993) in the discussion of Knox's paper: in a "lifestyle" investment strategy, the fund is invested predominantly in equities when the member is young and it is gradually switched into bonds and cash as the member approaches retirement.)

2. The model

Consider a defined contribution pension scheme where the fund can be invested in only two assets with different levels of risk, a low-risk asset and a high-risk asset. No decrements other than retirement are considered. Taxation and commission expenses are also not taken into account.

The proportion of the fund to be invested in the two assets is assumed to be reviewed every year, depending on the investment returns of the assets experienced and on the level of the fund compared with a specified target. This seems to be consistent with the real world, where the scheme member is informed every year about the growth of her/his position and can therefore modify the investment strategy, choosing the most appropriate solution in response to actual past experience. Therefore, the model we make is in discrete time.

Contributions are assumed to be paid yearly in advance. The contribution is assumed to be a fixed percentage of the salary, as in most defined contribution pensions schemes. The choice of defined contribution schemes follows recent laws in Italy in the context of occupational pension schemes.

The level of the fund at time t satisfies the following recurrence relationship:

$$f_{t+1} = (f_t + cS_t)[(1 - y_t)\,\mathrm{e}^{\mu_t} + y_t\,\mathrm{e}^{\lambda_t}], \tag{1a}$$

where f_t is the fund level at time t, c the contribution rate, S_t the salary at time t, y_t the proportion of fund invested in the high-risk asset during year $[t, t+1]$, μ_t the real force of interest for the low-risk asset in year $[t, t+1]$ which is assumed to be constant over the year $[t, t+1]$, λ_t the real force of interest for the high-risk asset in year $[t, t+1]$ which is assumed to be constant over the year $[t, t+1]$.

The assumption of a constant force of interest each year has been made on grounds of simplicity but also matches the annual review of asset allocation which is part of the model.

We use "real" to mean net of price inflation. Thus, the real rates of return for the two assets on year $[t, t+1]$ are $\mathrm{e}^{\mu_t} - 1$ and $\mathrm{e}^{\lambda_t} - 1$.

In this model, no real salary increase is considered and S_t is considered equal to 1 for simplicity, so that (1a) becomes

$$f_{t+1} = (f_t + c)[(1 - y_t)\,\mathrm{e}^{\mu_t} + y_t\,\mathrm{e}^{\lambda_t}]. \tag{1b}$$

It is assumed that the annual investment returns from the two assets are lognormally distributed, so that μ_t and λ_t are normally distributed. In addition $\{\mu_t\}$ and $\{\lambda_t\}$ are assumed to be sequences of independent and identically distributed random variables, so that

$$\mu_t \approx N(\mu, \sigma_1^2) \quad \text{and} \quad \lambda_t \approx N(\lambda, \sigma_2^2),$$

where

$$\mu \leq \lambda, \qquad \sigma_1^2 \leq \sigma_2^2.$$

It is also assumed that μ_t and λ_t are independent for any t. These assumptions are made on grounds of simplicity and we discuss their appropriateness in relation to the real world and related research in Section 5.

[3] In order to avoid confusion, it seems important to specify that in the insurance context, the term *annuity risk* is also referred to the risk that the insurer bears in paying the annuity to the policyholder until she/he survives. We do not use the term "annuity risk" with this meaning but in the sense explained above.