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Optimal investment strategies in the presence of a minimum guarantee☆

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Abstract

In a continuous-time framework, we consider the problem of a Defined Contribution Pension Fund in the presence of a minimum guarantee. The problem of the fund manager is to invest the initial wealth and the (stochastic) contribution flow into the financial market, in order to maximize the expected utility function of the terminal wealth under the constraint that the terminal wealth must exceed the minimum guarantee. We assume that the stochastic interest rates follow the affine dynamics, including the Cox–Ingersoll–Ross (CIR) model [Econometrica 53 (1985) 385] and the Vasiček model. The optimal investment strategies are obtained by assuming the completeness of financial markets and a CRRA utility function. Explicit formulae for the optimal investment strategies are included for different examples of guarantees and contributions.
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1. Introduction

Since Merton (1971), the optimal consumption-investment modelization is a central topic in mathematical finance. An important aspect is the introduction of a stochastic term structure for the interest rates. In the existing literature, a first stream of papers (e.g. Karatzas et al., 1987; Karatzas, 1989; El Karoui and Jeanblanc-Picqué, 1998) do not specify the stochastic process which leads to very general results. However, the general feature of the interest rates does not permit to test the results of those papers by comparing them with reality, since their solutions are not explicit.

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Afterwards, some authors focused on the possibility to obtain closed form solutions in order to test the behaviour of the optimal portfolio. For this reason, Bajeux-Besnainou et al. (1998, 1999) and Lioui and Poncet (2001) choose a Vasiček specification of the term structure.

In Deelstra et al. (2000), we investigated the case where interest rates follow the Cox–Ingersoll–Ross (CIR) dynamics. Assuming completeness of the markets and power utility function, we obtained by the use of the Cox and Huang (1989) methodology closed-form solutions for a utility maximization problem of terminal wealth, without considering contributions or a positive guarantee.

In this paper, we are interested in obtaining explicit optimal strategies for a defined contribution pension fund in the presence of a minimum guarantee in a continuous-time framework. Since this is a long-term investment problem of typically 30 or 40 years, it is crucial to allow for a stochastic term structure for the interest rates.

In the Vasiček framework, Jensen and Sørensen (2000) measure the effect of a minimum interest rate guarantee constraint through the wealth equivalent in case of no constraints and show numerically that guarantees may induce a significant utility loss for relatively risk tolerant investors.

Boulier et al. (2001) also study a problem closely related to ours, namely the optimal management of a defined contribution plan in the presence of a minimum guarantee. Nevertheless, in their framework, the contribution flow is a deterministic process, and the guarantee has a very specific form, which is an annuity paid out from the date of retirement until the date of death, where both annuity and date of death are supposed to be deterministic. Moreover, they choose the Vasiček specification of the term structure in the spirit of Bajeux-Besnainou et al. (1998, 1999).

We investigate the case in which interest rates follow the affine dynamics of Duffie and Kan (1996) in its one-dimensional version, which includes as special cases the CIR (Cox et al., 1985) model and the Vasiček (1977) model. Moreover, the problem of the fund manager is to invest the initial wealth and the stochastic contribution flow into the financial market, in order to maximize the expected utility function of the terminal wealth, which should exceed the minimum guarantee, a general random variable.

The paper is organized as follows: in Section 2, we define the market structure and introduce the optimization problem under consideration. We also show how this problem is related to the pension fund management. In Section 3, we transform the initial problem into an equivalent one, which we solve explicitely in the power utility case (Section 4). In Section 5 we come back to the solution of the initial problem and we find it explicitely by specifying the form of the contribution process in some interesting cases. Section 6 is devoted to numerical applications and Section 7 concludes the paper.

2. The model

In this section, we present:

(i) the financial market, that is the assets available and their equilibrium dynamics, given exogenously,
(ii) the optimization program, that is the characteristics of the agent and his optimization criteria.

We interpret also this modelization in terms of our pension fund problem.

2.1. The financial market

Randomness is described by a two-dimensional Brownian motion

$$\underline{z}(t) = \{(z(t), z_r(t))'; t \in [0, +\infty[\}$$

defined on a complete probability space $(\Omega, \mathcal{F}, P)$, where P is the real world probability. The filtration $\mathcal{F} = (\mathcal{F}_t)_{t\geq 0}$, represents the information structure generated by the Brownian motion and is assumed to satisfy the usual conditions.

Hereafter, $\mathbb{E}_t$ stands for $\mathbb{E}(\cdot|\mathcal{F}_t)$, the conditional expected value under the real world probability.

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