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Optimal consumption and investment strategies with a perishable and an indivisible durable consumption good

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Abstract

We study the consumption and investment choice of an agent in a continuous-time economy with a riskless asset, several risky financial assets, and two consumption goods, namely a perishable and a durable good with an uncertain price evolution. Assuming lognormal prices and a multiplicatively separable, isoelastic utility function, we provide an explicit Merton-type solution for the optimal strategies for the case where the durable (and all other assets) can be traded without transaction costs. For the case where the durable good is indivisible, in the sense that durable trades imply transaction costs proportional to the value of the current durable holdings, we show analytically that the optimal durable trading policy is characterized by three constants $\underline{z} < z^* < \bar{z}$. As long as the ratio z of the total current wealth to the value of current durable holdings of the investor is in $(\underline{z}, \bar{z})$, it is optimal not to trade the durable. At the boundaries of this interval it is optimal to trade the durable to attain $z = z^*$. The model is used to examine the optimal substitution between perishable and durable consumption and the importance of the durable price uncertainty and the correlation between the price of the durable good and financial asset prices.

JEL classification: G11; D11; D91; C61

Keywords: Optimal consumption and investment; Durable goods; Transaction costs; Viscosity solutions; Numerical solution

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1. Introduction

Modern economies offer an enormous variety of consumption goods. For modeling purposes each good is typically classified either as a *perishable* good or a *durable* good. A perishable good cannot be stored and provides utility only at the time of purchase. A durable good provides utility to its owner over a period of time and can be resold so that it also acts as an investment that transfers wealth over time. Traditional models of optimal consumption and investment problems consider either a single perishable consumption good, cf. Merton (1969), or a single durable consumption good, cf. Grossman and Laroque (1990). In this paper, we merge these settings by allowing for both a perishable and a durable good with a stochastically evolving relative price. This enables us to study optimal behavior in an economically more appealing setup and to address questions that cannot be dealt with in traditional single-good models, such as how optimal perishable and durable consumption policies are related and how the uncertainty about future relative consumption prices affects optimal consumption and investment decisions.

More specifically, we examine the optimal consumption and investment choice of an agent in a continuous-time economy with one riskless and several risky financial assets and both a perishable and a durable consumption good. The durable consumption good is indivisible in the sense that in order to change the stock of the good (beyond the assumed depreciation), the agent must sell his entire current holdings of the good and then buy the desired new stock, which is the case for houses and cars for example. We assume that in doing so the agent must pay transaction costs proportional to the value of the current stock of the durable. The perishable good and the financial assets are traded without transaction costs. The agent extracts utility from the rate of consumption of the perishable good and the stock of the durable good. We study the case where the agent has an infinite time horizon and a utility function of the multiplicatively separable, isoelastic form $U(c,k) = (c^{\beta}k^{1-\beta})^{1-\gamma}/(1-\gamma)$, where c and k are the current perishable consumption rate and the current stock of the durable, respectively. Furthermore, measured in terms of perishable consumption units, the price of the durable good and the prices of the risky financial assets follow correlated geometric Brownian motions.

Our first contribution is to derive an explicit solution to the consumption–investment problem for the case of no transaction costs. The optimal strategy is to keep both the perishable consumption rate, the value of the durable holdings, and the amount invested in each of the risky financial assets as fixed fractions of wealth. This result generalizes the solution to the single perishable consumption good problem of Merton (1969). The set of risky financial assets exhibits two-fund separation in that the optimal investment strategy combines only the mean-variance tangency portfolio and a durable hedge portfolio, which is the portfolio with the highest possible absolute correlation with the price of the durable consumption good.

The optimal strategy for the no transaction costs problem involves continuous rebalancing of the stock of the durable due to fluctuations in financial asset prices and the price of the durable and also due to the physical depreciation of the stock of the durable. With transaction costs, such a strategy is clearly not optimal. Our second contribution is to characterize the optimal consumption and investment policies with