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A risk-averse approach to simulation optimization with multiple responses

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ABSTRACT

This article considers risk-averse simulation optimization problems, where the risk measure is the well-known Average Value-at-Risk (also known as Conditional Value-at-Risk). Furthermore, this article combines Taguchi's robustness with Response Surface Methodology (RSM), which results in a novel, robust RSM to solve such risk-averse problems. In case the risk-averse problem is convex, the conic quadratic reformulation of the problem is provided, which can be solved very efficiently. The proposed robust RSM is applied to both an inventory optimization problem with a service-level constraint and a call-center problem; the results obtained for the risk-averse problem and its benchmark problem, where the risk measure is the variance, are intuitively sound and interesting.

1. Introduction

Robust Parameter Design (RPD) has been successfully applied to improve the quality of products since the mid 1980s, particularly after the work of G. Taguchi in US companies; see Taguchi [30,31]. In his RPD, Taguchi focuses on physical experiments, and he distinguishes between two types of variables, namely decision and environmental variables, among those variables that contribute to the experiments. His technique consists of determining the levels of the decision variables that reduce the sensitivity of the process to variations in the environmental variables, thus increasing the robustness of the process.

Although Taguchi's approach to RPD has become popular, his statistical techniques have received considerable criticism from many statisticians including Nair [22] and Myers et al. [20]; in particular, Vining and Myers [32] show inefficiency of Taguchi's signal-to-noise ratios under certain conditions; i.e., regression metamodels may provide statistically more rigorous alternatives to signal-to-noise ratios.

Classic Response Surface Methodology (RSM) focuses on the optimization of the mean of a single random response of industrial processes; see, for example, Myers et al. [20]. This approach is known to be risk-neutral, since the mean performance measure is optimized on average without taking into account, for example, its estimated variance. In RSM, this risk-neutrality problem is first detected by Myers and Carter [19], who introduce the Dual Response Surface (DRS) approach. The DRS approach is further extended by Vining and Myers [32]. Basically, in the DRS approach, two regression metamodels are fitted for the mean and the variance of a single random response, and then the two fitted regression metamodels are optimized simultaneously in a region of interest. Furthermore, the DRS approach uses Lagrange multipliers to explore candidate solutions in a manner similar to ridge regression.

The DRS approach has received a great deal of attention from researchers. Fan and Del Castillo [7] present a methodology for building a so-called optimal region for the global optimal operating conditions to address the inherent sampling errors in DRS. Ross et al. [28] extend the DRS such that the decision-makers' preferences can be incorporated into the selection of the

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Lagrange multipliers. Myers et al. [21] consider a general DRS in which the performance measure is a nonnormal response. Rajagopal et al. [24] provide a Bayesian approach to DRS. Lee and Park [17] obtain the robust optimal operating conditions using both fully and partially observed real-life data. Lee et al. [18] construct a robust design model which is resistant to contaminated data and to departures from the normality, and Köksoy and Yalcinoz [16] use a genetic algorithm with arithmetic crossover to solve their DRS problem.

Classic RPD assumes physical experiments. Then, experimental designs have been devised for physical settings, but in practical situations environmental variables are difficult, if not impossible, to control. Thus only a few of them, and with few levels, are usually included in the design. Recently, Dellino et al. [4] extend the robust RSM in Myers et al. [20] in many directions including the use of simulation outputs. This is an important contribution since simulation experiments enable the exploration of many values for the decision and environmental variables, and many combinations of these values. Therefore, the major purpose of this article is to contribute to the robust RSM methodology in Dellino et al. [4] in the following three directions:

- Dellino et al. [4] have a mean-risk approach where they consider standard deviation as their risk measure. The standard deviation (and variance), however, considers both the upper and lower deviations from the mean, whereas in many cases only one-sided deviation is important; see Yin et al. [34]. Hence, this article replaces the standard deviation by the *Average Value-at-Risk* (also known as the Conditional Value-at-Risk) from the financial engineering literature, which is known to consider only one-sided deviations. Furthermore, variance is considered as the risk measure of the benchmark problem.
- Dellino et al. [4] solve their risk-averse problem through MATLAB's `fmincon` function without investigating its convexity. This article, however, provides a very practical way to check for the convexity. Furthermore, in case the risk-averse problem is found to be convex, it provides a *conic quadratic reformulation*, which can then be solved very efficiently through, for example, CVX (a free-of-charge, structured convex programming solver).
- Dellino et al. [4] consider deterministic simulation whereas this article considers discrete-event dynamic (and hence stochastic) simulation. The use of stochastic simulation enables to observe many times the outputs of the multiple responses at the same input combinations through replicates, which is not meaningful for deterministic simulation. This is important since the average outputs of the multiple responses follow asymptotically a multivariate normal distribution because of the classic Central Limit Theorem, provided that the multiple responses have finite variances.

The proposed robust RSM is applied to the problem with the Average Value-at-Risk as its risk measure and the benchmark problem with variance as its risk measure on two classic cases, namely an (s, S) inventory optimization case with a service-level constraint and a call-center case, which are originally investigated by Bashyam and Fu [2] and Kelton et al. [11], respectively.

The remainder of this article is organized as follows. Section 2 introduces the problem formulation which considers Average Value-at-Risk as its risk measure. Section 3 presents the steps of the robust RSM; the emphasis in this section is on the optimization step where the benchmark problem and the conic quadratic reformulation are introduced. Section 4 gives numerical results of the risk-averse problem with the Average Value-at-Risk and its benchmark on the inventory optimization and the call-center optimization. Section 5 gives conclusions and possible future research directions.

2. The risk-averse problem formulation

This article considers the following optimization problem:

$$\begin{aligned} &\underset{\mathbf{d}\in R^k}{\text{minimize}} && \mathbb{E}[f_0(\mathbf{d},\mathbf{e},\boldsymbol{\omega})] \\ &\text{subject to} && \mathbb{E}[f_j(\mathbf{d},\mathbf{e},\boldsymbol{\omega})] \leqslant a_j \quad j=1,\ldots,r \end{aligned} \tag{1}$$

where $\mathbb{E}$ denotes the expectation operator, the f_i $(i = 0,\ldots,r)$ are $r + 1$ simulation outputs (responses), $\mathbf{d} = (d_1,\ldots,d_k)^T$ is the $k \times 1$ vector of decision variables, $\mathbf{e} = (e_1,\ldots,e_h)^T$ is the $h \times 1$ vector of environmental variables, $\boldsymbol{\omega}$ is the simulation's seed vector, and the a_j are r deterministic threshold levels. The multivariate distribution of $\mathbf{e}$ is assumed to be known (i.e., estimated from historical data) with mean vector $\boldsymbol{\mu}_\mathbf{e}$—which is not necessarily a zero-vector—and covariance matrix $\boldsymbol{\Sigma}_\mathbf{e}$—which is not necessarily a scaled identity matrix.

The following assumptions, which are common in black-box simulation optimization, are made for the problem in (1): (i) The f_i are continuous and continuously differentiable with respect to $\mathbf{d}$ for almost all $\boldsymbol{\omega}$ in the interior of the feasible region defined by the constraints. (ii) The mathematical formulations of the f_i are unknown to the decision-maker; at a given $\mathbf{d}$, $\mathbf{e}$, and $\boldsymbol{\omega}$, only their evaluations can be obtained through a simulation program. (iii) The expectations and the variances of the f_i are finite.

The drawback of the stochastic optimization problem in (1) is that it minimizes the random response f_0 on average, which is usually called a *risk-neutral* approach. A classic approach that takes risk into account is the expected utility theory, which minimizes $\mathbb{E}[u(f_0(\mathbf{d},\mathbf{e},\boldsymbol{\omega}))]$ where u is assumed to be a convex and nondecreasing disutility function. The obvious difficulty with this approach is to elicit an appropriate u.