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# Asymptotic and numerical analysis of the optimal investment strategy for an insurer

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**Abstract**

The asymptotic behaviour of the optimal investment strategy for an insurer is analysed for a number of cash flow processes. The insurer's portfolio consists of a risky stock and a bond and the cash flow is assumed to be either a normal or a compound Poisson process. For a normally distributed cash flow, the asymptotic limits are found in the cases where the stock is very risky or very safe. For a compound Poisson risk process, a composite asymptotic expansion is found for the optimal investment strategy in the case where stock is very risky and the claim size distribution is of an exponential type.

In general for a compound Poisson cash flow, the outer asymptotic limit reduces the integro-differential equation describing the optimal stock allocation to an integral equation, which determines the classical survival probability in ruin theory. The leading order optimal asset allocation is derived from this survival probability through a feedback law. Calculation of the optimal asset allocation leads to a difficult numerical problem because of the boundary layer structure of the solution and the tail properties of the claim size distribution. A second order numerical method is successfully developed to calculate the optimal allocation for light and heavy-tailed claim size distributions.
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## 1. Introduction

The net cash flow for an insurance portfolio from collected premiums and claims paid on insurance is called the risk process. Ruin theory has traditionally been the study of the probability of ruin of an insurer determined directly from the risk process without allowing for explicit investment of the insurer's surplus. Generally, an analytical expression for the ruin probability as a function of the current surplus (the ruin function) is only available for claim size distributions of exponential form. Rolski et al. (1999) develop the theory in considerable detail. As analytical results are not available research has concentrated on the behaviour of the ruin function as the current surplus tends to infinity. Lundberg bounds limit the size of the ruin function as long as the claim size distribution is light tailed. For sub-exponential claim size distributions the asymptotic behaviour of the ruin function as the surplus tends to infinity is proportional to the integrated tail distribution of the claims.

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We consider the optimal investment strategy for an insurer, which has a portfolio of a stock and a bond, where there is a stochastic cash flow arising from the risk process $R_t$. This problem was first analysed by Browne (1995) who adopted a normally distributed risk process. He found the optimal investment strategy for two different objectives: to minimise the probability of ruin and to maximise the utility of wealth for an exponential utility function. If there is no bond in the portfolio then for both these objectives it is optimal to invest a constant amount of money, independent of the current surplus, in the risky stock.

Hipp and Plum (2000) consider a portfolio consisting of just a risky stock, but use a compound Poisson process to model the risk process $R_t$. They found that the optimal strategy depends critically on the distribution of the claim sizes. In contrast to the normally distributed cash flow, the optimal strategy is to invest a fraction of the current surplus in the risky stock. Analytical results are available if the claim size distribution is exponential, and even here, an explicit optimal investment strategy can be found only for particular parameter choices. Liu and Yang (2004) extend the Hipp & Plum model to include a bond as well as a stock in the portfolio, and calculated the optimal asset allocation numerically for a number of different claim size distributions. A review of the optimal investment problem, as well as other optimal control problems in insurance, can be found in Hipp (2004).

There are many refinements to the basic model of ruin in the literature. Of relevance here is the work by Frolova et al. (2002) who consider the effect a *fixed* investment in a risky asset has on the ruin probability. They use an exponential claim size distribution and a Brownian motion for the risky asset and found the behaviour of the ruin probability for small and large volatility of the asset. Recent research for the optimal investment problem for an insurer has mirrored the development of ruin theory. Hipp and Schmidli (2003) determine the asymptotic behaviour of the ruin probability as the current surplus tends to infinity in the small claims case. Gaier and Grandits (2002) consider a claim size distribution with exponential moments, while Schmidli (2005) has determined the corresponding result for sub-exponentially distributed claim sizes.

Here, we study the asymptotic behaviour of optimal investment decision for an insurer using a small dimensionless parameter of the model rather than as the surplus tends to infinity. We use as this small parameter, the amount of risk in the asset defined by

$$\eta = \frac{\mu - r_0}{\sigma^2}, \tag{1}$$

where $\mu$, $\sigma$ are the drift and coefficient of volatility of the stock and $r_0$ is the risk free rate of interest. All these quantities are taken as constants. The drift $\mu$ and the risk free rate $r_0$ are usually quoted in units of percentage per annum. Thus they have dimension per unit time. If the stock is lognormally distributed, the coefficient of volatility $\sigma$ is also quoted per annum. This represents the observed standard deviation of the log return of the asset after one year, rather than signifying that this quantity has dimension per unit time. The coefficient of volatility has dimension per square root of unit time in order to be consistent with the dimension of the Brownian motion. Consequently, $\eta$ is a dimensionless quantity. Notice that with this notation the market price of risk is $\eta\sigma$.

Asymptotic methods have already been successfully applied to the optimal insurance pricing problem. Emms et al. (2006) use a perturbation expansion to determine the optimal premium in order to maximise the insurer's expected total wealth. This work represents an extension of those techniques to ordinary differential equations (ODEs) and integro-differential equations arising from the optimal investment problem. Hinch (1991) and Bender and Orszag (1978) develop the general theory of perturbation expansions in a small parameter to give approximate analytical solutions.

In Section 2 we identify the limiting behaviour of the optimal strategy as $\eta \to 0$ for a normally distributed risk process. Using this feature of the model allows us to construct approximate optimal strategies when the parameter is not zero, and so describe the qualitative features of the optimal strategy when analytical results are not available. The cash flow is taken as compound Poisson process in Section 3. We obtain approximate optimal strategies for two light-tailed claim size distributions in Section 4. For these distributions the problem reduces to the solution of an ODE and a system of ODEs. It is easy to integrate these equations numerically so the exact solution can be readily compared with the asymptotic results. For some of the analysis, asymptotic results are only available analytically if there is no bond in the portfolio. We examine this case by setting $r_0 = 0$. Section 5 develops the theory for general claim size distributions and describes three numerical schemes to solve the optimal asset allocation problem when there is no reduction to ODEs. Conclusions are given in Section 6.

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