The Quarterly Review of Economics and Finance 50 (2010) 167–179

Contents lists available at ScienceDirect

The Quarterly Review of Economics and Finance

journal homepage: www.elsevier.com/locate/qref

Behavior patterns of investment strategies under Roy's safety-first principle[☆]

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ARTICLE INFO

Article history:
Received 22 January 2009
Received in revised form 30 October 2009
Accepted 3 November 2009
Available online 16 December 2009

JEL classification:
G11
D81

Keywords:
Continuous-time portfolio selection
Safety-first principle
Investment strategies
Horizon effects
Efficient frontier

ABSTRACT

The safety-first principle is a natural motivational factor in decision making, and is closely related to certain popular heuristics such as *satisficing*. We provide a systematic analysis of optimal portfolio choice under Roy's safety-first principle by examining and comparing the behavior patterns of three popular investment strategies: the optimal constant-rebalanced portfolio, dynamic-rebalanced portfolio and buy-and-hold strategies. Our results indicate the importance of a match between the investment strategy, the investment goal, and the investment horizon. We also develop a geometric approach to investigate the relationships among the safety-first, expected utility, and mean-variance models and offer an explanation for the long-standing debate concerning different patterns of time-diversification effects.

1. Introduction

Establishing appropriate performance criteria for investment decisions and deriving the corresponding best portfolio policies are two interrelated and fundamental tasks that must be accomplished when investing in financial markets. As indicated in Markowitz (1999) and widely accepted by the research community, the development of modern portfolio theory has its origin in two seminal papers published in 1952: Markowitz (1952) and Roy (1952). In the first pioneering work, Markowitz (1952) develops the well-known mean-variance (MV) approach under which investors have to achieve a balance between two conflicting objectives: maximizing the expected return of a portfolio and minimizing the investment risk measured by the portfolio variance. In the second pioneering work, Roy (1952) proposes the so-called safety-first principle, which suggests that investors behave in such a way that the probability of the portfolio value falling below a specified disaster level is minimized,[1] i.e., the investment objective under Roy's safety-first principle is to minimize the ruin probability or maximize the chance of survival. Mathematically, the safety-first principle can be formulated in the following way:

$$\min \quad \alpha := P(\tilde{W} < C),$$

where $\tilde{W}$ is the (random) final wealth, C is the so-called subsistence or disaster level and α is the ruin probability.

As the natural choice in many situations, Roy's safety-first approach laid the foundation for many later developments in finance, most prominently those in behavioral finance and risk management. In a certain sense, the subsistence level in the safety-first principle can also be viewed as the satisfaction level. Simon (1955), who introduces the term bounded rationality (also called

☆ We are very grateful for the review comments and suggestions from an anonymous referee, which facilitate a significant improvement of the paper. The research work of the first author is partially supported by the National Science Foundation for Distinguished Young Scholars (No. 70825002), the National Natural Science Foundation of China (No. 70518001), and the National Basic Research Program of China (973 Program, No. 2007CB814902), the research work of the second author is partially supported by Shanghai Pujiang Program, and the research work of the third author is partially supported by the Research Grants Council of Hong Kong under RGC4245/04E, (N_CUHK445/05) and CUHK414808.
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[1] It is worth mentioning that there are other safety-first rules (see, e.g., Telser, 1956; Kataoka, 1963) in the literature. We consider in this paper, however, only the original form of the safety-first rule, Roy's safety-first rule. The "safety-first principle" mentioned throughout thus refers to "Roy's safety-first principle".

1062-9769/$ – see front matter
doi:10.1016/j.qref.2009.11.004

limited rationality), suggests that individuals should simplify their decision problems using the satisficing approach: satisfactory if the target is reached or unsatisfactory if it is not. Hence, Roy's safety-first principle can also serve as a behavioral criterion for rational choices. A notable application of this principle in behavioral finance can be found in the work of Shefrin and Statman (2000). The most widely adopted risk measure in risk management, Value-at-Risk (VaR) (see, e.g., Duffie & Pan, 1997; Jorion, 1997), and many other downside risk measures are rooted in Roy's safety-first principle (see, e.g., Basak & Shapiro, 2001; Bawa, 1976, 1978; Nawrocki, 1999).

The past decade or so has witnessed a number of studies related to this safety-first principle. For example, Roy (1995) develops a discrete-time dynamic optimization problem in which the objective is to maximize the long-run probability of survival. Li, Chan and Ng (1998) solve a multi-period safety-first portfolio selection problem by converting it into a corresponding mean-variance formulation using the Chebychev inequality. Adopting a continuous-time market setting, the papers by Browne (1995, 1997, 1999a,b,c) derive the optimal dynamic investment strategy for a portfolio manager who seeks to maximize the probability of reaching a particular goal.[2] Föllmer and Leukert (1999) introduce the safety-first concept into dynamic hedging problems. The papers by Stutzer (2000, 2003) draw on this concept to propose a new portfolio selection criterion that maximizes the decay rate of the probability of realizing a portfolio return below some predetermined target return level. Chiu and Li (2009) extend Roy's safety-first principle to the asset-liability management problem. It should be noted that most of the above mentioned papers deal with infinite horizon problems, rather than those with a finite time horizon. Furthermore, compared to the study of dynamic mean-variance portfolio selection (see, for example, Bielecki, Jin, Pliska, & Zhou, 2005; Li & Ng, 2000; Lim & Zhou, 2002; Yin & Zhou, 2004; Zhou & Li, 2000; Zhou & Yin, 2003), a systematic analysis of different types of investment policies under the safety-first principle remains lacking.

We thus devote this paper to a systematic investigation of investment decision behavior patterns under Roy's safety-first principle. To provide a comprehensive analysis of this principle, this paper examines and compares the three most popular portfolio selection strategies adopted in both academic study and real investment practice (see, e.g., Brandt, 1999; Barberis, 2000): the buy-and-hold, the dynamic-rebalanced portfolio (DRP) and constant-rebalanced portfolio (CRP) strategies. In a buy-and-hold strategy, the determination of a portfolio occurs only at the beginning of the investment horizon. A DRP strategy allows investors with maximum freedom to dynamically change the weights of the individual securities in the portfolio over time. A CRP strategy, in contrast, retains the same distribution of wealth among the securities all of the time; that is, the proportion of the investor's total wealth that is invested in each of the underlying securities remains the same all of the time. It should be noted that such a CRP strategy still implies continuous-trading, as one has to trade at every time instant to ensure that the investment proportions are rebalanced back to their original settings as the stock prices move up and down. A variety of optimality properties have been found for the CRP policies in ordinary portfolio selection, and these have been widely used in asset allocation practice (see, e.g., Perold & Sharpe, 1988).

[2] Roy's safety-first principle can equivalently be described as maximizing the probability of achieving a certain goal, whereby the subsistence level corresponds to the investment goal.

In this paper, we examine how different types of investment strategies work under the safety-first principle. Within our framework, there are three dimensions for the portfolio choice problem: the investment strategy, the investment horizon and the investment goal. By comparing the optimal solutions of the different strategies, we intend to address an issue that in which situation a certain investment strategy confers a relative advantage.

We also establish the relationships between the time-diversification effect (i.e., the horizon effect) and the investment's target growth rate based on the optimal portfolio choice under the safety-first principle. Different patterns exist for the horizon effect in the financial literature. In spite of the vast amount of work carried out on the topic of the horizon effect, controversy remains about its implications for optimal asset allocations. Prevalent in the investment world is a belief in the time-diversification effect: the longer the investment horizon, the more that an investor should invest in stocks. However a number of empirical studies support the existence of a reverse time-diversification effect. For example, Ameriks and Zeldes (2001) report that, ceteris paribus, elderly people typically hold more risky positions than do younger people. However, these opposite effects cannot be justified by risk aversion in the traditional expected utility framework. Merton (1969, 1971) and Samuelson (1969) both find that if asset returns are of the nature of a random walk, then the optimal strategy for an investor with a Constant Relative Risk Aversion (CRRA) utility is independent of the investment horizon, i.e., there is no horizon effect in determining the asset allocation. In this paper, we demonstrate that different horizon effect patterns can be intuitively understood within the safety-first framework. More specifically, our results show that the investment decisions generated under the safety-first principle with different subsistence level patterns demonstrate different horizon effect patterns. For example, a subsistence level that grows at a faster rate than the exponential rate yields the time-diversification effect. By considering both the investment goal and the investment horizon under the safety-first principle, we offer an explanation for the long-standing debate concerning the different time-diversification effects.

The contributions of the research presented in this paper can be summarized as follows.

- We provide a rather detailed examination of the safety-first portfolio selection problem from the perspective of three basic trading strategies: the buy-and-hold, DRP and CRP strategies.
- We investigate the relative advantages and disadvantages of active and passive trading behaviors under the safety-first principle.
- We develop a geometric approach to investigate the relationships among the safety-first, expected utility and mean-variance models under a continuous-time setting.
- We establish the relationship between the time-diversification effects and an investment's target growth rate within the framework of the safety-first principle.

The reminder of the paper is organized as follows. In Section 2, we introduce our problem setting under the safety-first principle in a Black–Scholes continuous-time market. In Section 3, we summarize the closed-form expressions for the three types of optimal strategies under the safety-first formulation. We extend the results assuming a constant subsistence level to a problem with a dynamic subsistence level in Section 4. In Section 5, we contrast the three types of optimal strategies by comparing their terminal wealth distributions under different performance measures, thus revealing some of the rich properties of the safety-first principle. Motivated by the research presented in Pyle and Turnovsky (1970, 1971) in a one-period context, we devote Section 6 to the development of