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Optimal investment strategy for the DC plan with the return of premiums clauses in a mean–variance framework

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HIGHLIGHTS

- Establish a new continuous-time risk model in the DC pension plan.
- Study optimal investment strategy for the DC plan with return of premiums clauses.
- Give an optimal strategy and the efficient frontier of the pension member in a mean–variance framework.

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ABSTRACT

In this paper, we study the optimal investment strategy in the DC pension plan during the accumulation phase. During the accumulation phase, a pension member contributes a predetermined amount of money as premiums and the management of the pension plan invests the premiums in equities and bonds to increase the value of the accumulation. In practice, most of the DC pension plans have return of premium clauses to protect the rights of the plan members who die during the accumulation phase. In the model, the members withdraw their premiums when they die and the difference between the premium and the accumulation (negative or positive) is distributed among the survival members. From the surviving members' point of view, when they retire, they want to maximize the fund size and to minimize the volatility of the accumulation. We formalize the problem as a continuous-time mean–variance stochastic optimal control problem. The management of the pension plan chooses the optimal investment strategy, i.e., the proportions invested in equities and bonds, to maximize the mean–variance utility of the pension member at the time of retirement. Using the variational inequalities methods in Björk and Murgoci (2009), we transform the mean–variance stochastic control into Markovian time inconsistent stochastic control, then establish a verification theorem, which is similar to one of He and Liang (2008, 2009) and Zeng and Li (2011), to find the optimal strategy and the efficient frontier of the pension member. The differences of the optimal strategies between the Pension plans with and without the return of premium clauses are studied via the Monte Carlo methods. The impacts of the risk averse level on the optimal strategies is also explored by the numerical methods.

1. Introduction

The defined contribution (DC) pension plan plays an important role in the social security system, and it has become popular in the pension market due to the demographic evolution and the development of equity markets. In the DC pension plan, a member contributes a predetermined amount of money as premiums during the accumulation phase. When the member retires, the accumulation will be distributed monthly as old-age pension. The distribution is not predetermined, but depends on the mortality risk, inflation and investment efficiency, etc. The retirement benefits of the DC pension plan depends on the returns of the fund's portfolios, so the asset allocation decisions are crucial to DC pension management. In this paper, we study the optimal control problem on the asset allocation strategies during the accumulation

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phase of the DC pension plan. The accumulation phase lasts for the whole working period of the pension member. The accumulation is invested in equities and bonds by the pension management to increase the fund size. A lot of literature on the accumulation phase of the DC pension plan focuses on optimal portfolio selection problems according to some criterion. Cairns (2000), Battocchio and Menoncin (2004) and He and Liang (2013) study the stochastic optimal control problem often DC pension fund in continuous time. Boulier et al. (2001) study optimal investment strategies of the DC pension plan under the stochastic interest rate. Haberman and Vigna (2002) study the optimal investment strategy and risk measures of the DC pension plan. Han and Hung (2012) first study the optimal dynamic asset allocation for DC pension plans with stochastic interest rates and inflation in a continuous-time model.

Referring to the objective function aspect, there are two types of wildly used utility goals. One is to maximize the accumulation at the time of retirement and the other is to balance the return and the risk, i.e., maximizing the fund size and minimizing the volatility of the accumulation. The former goal includes three types of utility functions: the utility function which exhibits constant relative risk aversion (CRRA), the utility function which exhibits constant absolute risk aversion (CARA), and quadratic loss functions. Literature concerned with the CRRA utility function are Cairns et al. (2006) and Gao (2008). They choose the power or logarithmic utility function as the objective function. Literature concerned with the CARA utility function are Devolder et al. (2003) and Battocchio and Menoncin (2004). They choose the exponential utility function as the objective function. There are also some utility functions which are not widely used. Haberman and Vigna (2002) minimize the quadratic loss function and Di Giacinto et al. (2011) use a general form of utility function to study the optimal control policies during the accumulation phase of the DC pension plan. The latter goal includes the mean–variance utility and the value-at-risk (VaR) utility, etc. The mean–variance utility originates from Markowitz (1952) and Richardson (1989), which study the single period investment allocation problem under the mean–variance utility. Richardson (1989), Bajeux-Besnainou and Portait (1998) and Zhou and Li (2000) extend the single period mean–variance stochastic control to continuous time models and find the efficient frontiers. The literature on the optimal control problem of the DC pension plan is Højgaard and Vigna (2007). Vigna (2009) compares the mean–variance efficiency of the CARA and CRRA utility in the DC pension plan. The problem with VaR utility function equals the problem of maximizing the terminal accumulation with the minimal guarantee. Li et al. (2007) study the mean-CaR problem. In this paper, the goal of individual pension member is to maximize the fund size and to minimize the volatility of the accumulation. We choose the mean–variance utility as our criterion. The whole problem could be formalized into a continuous time mean–variance optimal stochastic control one. The management of the pension plan chooses the optimal investment strategies, i.e., the proportions allocated in equities and bonds, to maximize the mean–variance utility of the pension member at the time of retirement.

In this paper, we study the optimal control problem of the DC pension plan with the return of premiums clauses. In order to protect the rights of the plan members who die early during the accumulation phase, most of the DC pension plans have return of premium clauses. In this kind of actuarial clause, the dead member can withdraw the premiums she/he contributes or the premiums accumulated by a predetermined interest rate. For simplicity, we study the former one in our model. After returning the premium, the difference (positive or negative) between the return and the accumulation will be equally distributed by the surviving members. It means that the surviving members will stand the mortality risk and the investment efficiency risk of the pension fund. Taking the above actuarial rules into consideration in our model is the main contribution of our paper. Blake et al. (2003) and Milevsky and Robinson (2000) transform the actuarial clauses of the DC pension plan during the distribution phase into a discrete time problem. With the help of these papers, we extend the problem with the return of premium clauses into a continuous-time mean–variance stochastic control problem. Using the variational inequalities methods in Björk and Murgoci (2009), we transform the mean–variance stochastic control into Markovian time inconsistent stochastic control, then, in the same way as one of He and Liang (2008, 2009), Liang and Huang (2011) and Zeng and Li (2011), we establish a verification theorem to give the optimal proportions allocated in the equities and the bonds and find the efficient frontier of the pension member. In the paper, it is the first time we study the optimal utility problem of the individual pension member and the accumulation of the individual member's account, which is firstly described by a continuous-time stochastic process according to the actuarial rules. The differences of the optimal strategies between the Pension plans with and without the return of premium clauses are studied using Monte Carlo methods.

The paper is organized as follows: In Section 2, we introduce the actuarial methods of the DC pension plan with the return of premiums clauses. Under the return of premiums clauses, the survival members stand the mortality risk and the investment efficiency risk as well. In this section, we formalize the fund size process as the solution of a continuous time stochastic differential equation. In Section 3, we use the mean–variance utility as the criterion of the pension members. The survival member wants to maximize the fund size and to minimize the volatility of the accumulation. Using the variational inequalities methods in Björk and Murgoci (2009), He and Liang (2008, 2009), Liang and Huang (2011) and Zeng and Li (2011), we establish the optimal proportions allocated in the equities and the bonds, and the efficient frontier of the pension members. In Section 4, the impacts of the risk aversion coefficient on the optimal investment strategy are given via numerical examples. The differences of the optimal strategies between the pension plans with and without the return of premium clauses are studied using Monte Carlo methods. Summarizing comments are listed in the last section.

2. The continuous-time model of the DC pension plan with return of premiums clauses

In this paper, we study the optimal investment policy of the DC pension plan with the return of premiums clauses during the accumulation phase. In the DC pension plan, the member contributes a predetermined amount of money as premiums during the accumulation phase. In the model, we suppose the premium per unit time is P, which is a predetermined variable. The accumulation phase lasts the whole working period of the pension member. We suppose the accumulation period starts from the age of ω_0 and lasts to the age of $\omega_0 + T$, i.e., the time length of the pension fund is T. During the phase, the premium is invested in equities and bonds by the pension management to increase the value. The proportion allocated in the equities is π, which is a control variable. The remaining $1 - \pi$ is allocated in bonds. When the pension member retires, she/he will get old-age pension from the fund; the amount is not predetermined and it is affected by the uncertainty of the mortality risk and the investment efficiency. In order to protect the rights of the plan members who die early, i.e., during the accumulation phase, most DC pension plans have return of premiums clauses. In this kind of actuarial clause, the dead member can withdraw the premiums she/he contributes or the premiums accumulated at a predetermined interest rate. For simplicity, we study the former one in our model. In the model, ${}_{\frac{1}{n}}q_{\omega_0+t}$ is the mortality rate from time t to time $t + \frac{1}{n}$, and tP is the accumulated premium at time t. So, the premium returned to the dead member from time t to time