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Incomplete markets, ambiguity, and irreversible investment

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ABSTRACT

The problem of irreversible investment with idiosyncratic risk is studied by interpreting market incompleteness as a source of ambiguity over the appropriate no-arbitrage discount factor. The maxmin utility over multiple priors framework is used to model and solve the irreversible investment problem. Multiple priors are modeled using the notion of κ-ignorance. This set-up is used to analyze finitely lived options. For infinitely lived options the notion of constant κ-ignorance is introduced. For these sets of density generators the corresponding optimal stopping problem is solved for general (in-)finite horizon optimal stopping problems driven by geometric Brownian motion. It is argued that an increase in the set of priors delays investment, whereas an increase in the degree of market completeness can have a non-monotonic effect on investment.

1. Introduction

In the real options literature it has been established that as long as the risk underlying an irreversible investment project is perfectly spanned by traded assets it is relatively straightforward to obtain an appropriate discount factor for valuing the project (cf. Thijssen, 2010). If there is only partial spanning, however, there is not enough pricing information in the market to obtain a unique discount factor to appropriately price all risk in the project using the no-arbitrage condition alone. Instead, there are infinitely many discount factors that price the market (in the sense that no-arbitrage opportunities arise). In such cases markets are incomplete and the option is non-redundant. Most of the current literature on irreversible investment then resorts to a preference-based approach. That is, risk preferences are specified for the decision-maker and – hence – the problem of market incompleteness is circumvented entirely. See, for example, Miao and Wang (2007) and Hugonnier and Morellec (2007). A notable exception is Henderson (2007) who incorporates the Sharpe ratio of a partial spanning asset in an expected utility optimization problem with CARA preferences. She assumes that the owner-manager maximizes expected utility taking into account the (partial) hedging opportunities that are present in the market.

In this paper I argue that market incompleteness is, in essence, a case of incomplete or imperfect information, since the decision-maker cannot extract enough information from the price signal to make an unambiguous investment decision. Since investment decisions should be taken in the interest of the stockholder, the decision-maker, therefore, faces ambiguity over the correct way in which the cash-flows accruing from the project have to be discounted.

CAPM tells us that the manager should consider this non-marketed risk as idiosyncratic and, therefore, not take it into account when appraising the investment project.[1] This approach has been used throughout the literature (see, for example, Dixit and Pindyck, 1994). If, however, the firm is publicly listed, then the idiosyncratic risk does become marketed as soon as investment in the project takes place. At that stage the market will price this risk. In an ideal world the manager should

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[1] This result arises due to the assumption of quadratic utility. It survives with CARA utility, but, for example not for investors with CRRA preferences.

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already take this market price into account when finding the appropriate rate at which to discount the revenue stream accruing from the project.

In recent years there has been renewed interest in the difference between ambiguity and risk. The distinction was made most clearly by Knight (1921), but lay dormant for many years while (subjective) expected utility theory was developed and took center-stage in decision theory under risk. Two recent paradigms for dealing with ambiguity as opposed to risk have been developed by Gilboa and Schmeidler (1989) and Bewley (2002). Both approaches are aimed at modeling ambiguity-aversion, for which experimental evidence has been found ever since the Ellsberg paradox (cf. Ellsberg, 1961). The former approach assumes that a decision-maker has multiple priors and chooses the prior that maximizes the minimal payoff over acts, so-called maxmin utility. The latter approach postulates that decision-makers start from a status quo and will only choose actions that are preferred to this status quo in all states of the world.

Following Nishimura and Ozaki (2007) and Trojanowska and Kort (2010) I use the multiple priors maxmin utility framework as devised by Gilboa and Schmeidler (1989) to analyze irreversible investment decisions under ambiguity. In order to build continuous-time models the specification of the multiple priors model introduced by Chen and Epstein (2002) is applied. It is important to note that, whereas Nishimura and Ozaki (2007) and Trojanowska and Kort (2010) assume that there is ambiguity over certain characteristics regarding the future cash-flows of the project, this paper assumes that these are purely risky. Here, the ambiguity is solely about the appropriate rate at which the cash-flows should be discounted. It turns out, however, that under a particular form of ambiguity over the discount factor, the problem is essentially the same as that of ambiguity with respect to the cash-flows in Nishimura and Ozaki (2007) and Trojanowska and Kort (2010). In both cases, namely, the presence of ambiguity leads to a change in the probability measure under which the decision-maker takes the investment decision. This change in measure relies on Girsanov's theorem.

In particular, the special case where the cash-flows follow a geometric Brownian motion and where ambiguity takes the form of κ-ignorance is analyzed numerically. Under κ-ignorance, at any time the market price for risk essentially lies in a symmetric bounded interval around a reference prior. The advantage of this form of ambiguity is that it is entirely described by one parameter, which makes comparative statics easier. The numerical example suggests that, *ceteris paribus*, ambiguity delays investment. Market incompleteness, however, can have a non-monotonic effect on investment, depending on how it is defined and the time horizon of the option. If the degree of market incompleteness is measured by the volatility of the non-marketed risk, then with a finite horizon there can be a non-monotonic effect of market incompleteness on the optimal investment policy. This happens because of two opposing effects. On the one hand there is an *option effect* due to total volatility going up. This increases the value of the option and, thus, increases the optimal investment threshold. Secondly, there is an effect that I call the *no-arbitrage effect*, which arises because an increase in non-marketed risk decreases the correlation between the partial spanning asset and the project's payoffs. This effect is non-monotonic and in the case of a finitely lived option it can dominate.

If one restricts the class of priors to a smaller set of density generators than those representing κ-ignorance I show that it is possible to analyze infinitely lived options analytically. The advantage of this is that the optimal investment policy takes the familiar form of a constant trigger: invest as soon as the underlying uncertainty reaches a certain trigger from below. The simplicity of the optimal policy, in turn, implies that analytical results on comparative statics can be obtained. It turns out that ambiguity delays investment also in the infinite horizon case. In addition, the non-monotonicity of the no-arbitrage effect of an increase in market incompleteness, as measured through a decreasing correlation between marketed and non-marketed risk, survives. However, the option effect increases in such a way that, on balance, the investment threshold increases unambiguously in the volatility of non-marketed risk. This shows that moving from a finite to an infinite horizon is not innocuous.

Qualitatively, the result that investment is delayed by ambiguity is also obtained in Nishimura and Ozaki (2007) and Trojanowska and Kort (2010), although in a different setting. The quantitative effect, however, will be different because, in this paper, ambiguity is restricted to that part of the project's payoffs that is not spanned by the market.

The paper is organized as follows. In Section 2 the problem of irreversible investment under incomplete markets is stated. In Section 3 the multiple priors model for ambiguity is introduced and applied to the irreversible investment problem. An analysis of the optimal investment policy for finite and infinite time horizons under geometric Brownian motion is conducted in Sections 4 and 5, respectively. Section 6 concludes.

2. Irreversible investment in incomplete markets

Suppose that a firm can invest in an irreversible project. Let $T \in (0,\infty]$ be the life-time of this option. The value of the firm, resulting from the project is risky and is modeled as follows. Let $(\Omega,\mathcal{F})$ be a measurable space, endowed with a filtration $(\mathcal{F}_t)_{0 \le t \le T}$, such that $\mathcal{F} = \mathcal{F}_T$. For all $\nu > 0$, let P_ν be a probability measure on $(\Omega,\mathcal{F})$. The project's value is assumed to follow an Ito diffusion of the form

$$\frac{dV_t}{V_t} = \mu\, dt + \sigma_z\, dz_t + \sigma_w\, dw_t, \tag{1}$$

where $\mu \in \mathbb{R}$, $\sigma_z > 0$ and $\sigma_w > 0$ are constants, and $(z_t)_{t \ge 0}$ and $(w_t)_{t \ge 0}$ are independent P_ν-Brownian motions, such that $V_0 = \nu$, P_ν-a.s. The instantaneous volatility of V is denoted by σ, that is, $\sigma^2 = \sigma_z^2 + \sigma_w^2$. The net present value of the project is assumed to be a C^2 function on the state space, $F : \mathbb{R}_+ \to \mathbb{R}$, with $F' > 0$, $F'' \le 0$, and $F(0) < 0$.