PDE models and numerical methods for total value adjustment in European and American options with counterparty risk

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Abstract

Since the last financial crisis, a relevant effort in quantitative finance research concerns the consideration of counterparty risk in financial contracts, specially in the pricing of derivatives. As a consequence of this new ingredient, new models, mathematical tools and numerical methods are required. In the present paper, we mainly consider the problem formulation in terms of partial differential equations (PDEs) models to price the total credit value adjustment (XVA) to be added to the price of the derivative without counterparty risk. Thus, in the case of European options and forward contracts different linear and non-linear PDEs arise. In the present paper we propose suitable boundary conditions and original numerical methods to solve these PDEs problems. Moreover, for the first time in the literature, we consider XVA associated to American options by the introduction of complementarity problems associated to PDEs, as well as numerical methods to be added in order to solve them. Finally, numerical examples are presented to illustrate the behavior of the models and numerical method to recover the expected qualitative and quantitative properties of the XVA adjustments in different cases. Also, the first order convergence of the numerical method is illustrated when applied to particular cases in which the analytical expression for the XVA is available.

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1. Introduction

Since 2007 crisis, when important financial entities went bankrupt, the counterparty risk has become an important ingredient that needs to be taken into account in all financial contracts. It can be described as the risk to each party of a contract that the counterparty will not live up to its contractual obligations. Different institutions and financial analysts consider that the crisis was due to the mistakes made in the financial system, namely in the management of the risk. The complexity of the financial derivatives and the consideration of a low probability of default were two of the factors that led to the crisis. As a consequence, a review of the counterparty risk consideration has been addressed.

From the point of view of the seller, the risk neutral value of a derivative can be currently adjusted by the following items [25]:

- It is reduced by the existence of funding costs, in the case the latter takes part (FCA).
- It is increased in the case its value produces liquidity for the entity (FBA).

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• It is reduced by the necessary costs to compensate the credit risk due to the counterparty (CVA).
• If a bilateral counterparty risk is assumed, the derivative value is increased by its potential benefits due to the issuer probability of default and the issuer has not to face its contractual responsibilities, when those are positive for the issuer (DVA).
• It is increased by the cost of borrowing the collateral (CollIVA).

The FCA and the FBA can be merged and the sum of them is known as FVA (funding valued adjustment), understood as the correction to the risk-free price to account for the funding costs. The presence of FVA in the adjustment is reasonable in the case of non-collateralized trades; however when a collateral is posted to fully cover the counterparty risk then the FVA reduces to zero. In this sense, FVA is given by the difference of price between non-collateralized and fully collateralized contracts (see [24]). CVA represents the price to mitigate counterparty credit risk on a trade and the concept was first introduced in [12,18,26]. However, as no parts in the contract are risk-free, then DVA is the price of the hedging used to mitigate the own credit risk, so from the other counterparty is understood as a CVA. DVA was first introduced in [12] to account for the presence of two risky counterparties and the consideration of DVA allows to agree on the price by both traders (symmetric prices). However, a long controversy exists about the consideration of DVA and the same happens with FVA (see [7,14–16,20] for different views on FVA).

Thus, including counterparty risk in the pricing of derivatives represents an important change in the existent risk-free pricing models. In particular, in this setting nonlinear partial differential equations (PDE) models can be posed, which have to be mathematically analyzed and solved by means of suitable numerical methods. The main goal of the present paper concerns the computing of the European and American options price, accounting for all the associated cash flows that come from the derivative itself, the act of hedging, the default risk management and the funding costs. Following the usual terminology, we will refer to the total value of these adjustments as XVA, which in terms of the previously introduced notations is defined by:

\[ XVA = DVA - CVA - FCA + FBA + CollIVA = DVA - CVA + FVA + CollIVA \]

So, we pose the PDE models for the derivative value, \( \hat{V} \), from the point of view of the seller, when the trade takes place between two risky counterparties. More precisely, we focus on the case of European and American vanilla options. We use hedging arguments to derive the extensions to the Black–Scholes PDE in the presence of bilateral jump-to-default model and include funding considerations into the financing of the hedge positions.

Actually, nowadays there are three main methodologies to include funding costs, collateral and credit risk in the pricing of derivatives. A first approach, following the seminal papers by Piterbarg [24] and Burgard and Kjaer [4] that obtain PDE formulations by means of suitable hedging arguments and the use of Ito lemma for jump-diffusion processes. In [24] funding costs are introduced while in [4] both funding costs and bilateral counterparty credit risk are considered. This approach is also followed in [13] in the more general setting of stochastic spreads, in which three underlying stochastic factors are involved. Moreover, in [13] the solution is also equivalently written in terms of expectations. A second approach follows the initial ideas in [2] to include DVA by means of expectations, next extended to the collateralized, close-out and funding costs in [22]. A third approach is based on backward stochastic differential equations introduced in [9,10]. In all previous papers, the case of European derivatives is addressed.

In the present paper we follow the first approach in the line of Burgard and Kjaer [4] and propose original numerical methods for solving the PDE models. Thus, after recalling the hedging strategy proposed in this paper of the case of European-style derivatives, different kinds of PDEs arise depending on the assumption of the mark-to-market value at default. Thus, if this mark-to-market value is equal to the riskless derivative then a linear PDE that involves the value of the riskless derivative is obtained. However, if the mark-to-market value is given by the risky derivative, then a nonlinear PDE is obtained. In the linear case, the equivalent expression of the solution in terms of expectations can be solved. In the nonlinear case, this equivalent expression takes the form of a nonlinear integral equation and numerical methods are also required. In the present paper we propose a set of numerical techniques to solve the resulting PDEs for both choices of the mark-to-market at default. For this purpose, we truncate the unbounded asset domain and pose original suitable boundary conditions at the boundaries of the resulting bounded domain, following some ideas in [8] also taken from [11]. After truncation, we propose a time discretization based on the method of characteristics combined with a finite elements discretization in the asset variable. For the case leading to a nonlinear PDE a fixed point iteration algorithm is proposed.

Another original point is the consideration of American-style options. In this case, previously we have to solve numerically the associated obstacle problems as an additional nonlinearity. For this purpose we use an augmented lagrangian active set (ALAS) algorithm proposed in [19], already used in [1,6] for problems related to investment valuation and pension plans with early retirement opportunity, respectively.

The plan of the paper is the following. In Section 2, some one stochastic factor models in the literature to price European-style options in the presence of counterparty credit risk are described. More precisely, first counterparty credit risk and funding costs are considered, while in a second step the collateral is added to the previous model. In Section 3 we incorporate original models to price American-style options when XVA is considered. Section 4 is devoted to the description of different numerical methods that are proposed to solve the linear and nonlinear PDE models stated in Section 2. Particularly, the domain truncation to pose the PDE problem in a bounded domain requires the consideration of appropriate and original boundary conditions. In Section 5 we present and discuss the numerical results for different examples. Finally, some conclusions are indicated.
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