Available online at www.sciencedirect.com
ScienceDirect
Journal of Economic Theory 136 (2007) 668–694
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Irreversible investment and Knightian uncertainty☆

Kiyohiko G. Nishimura[a], Hiroyuki Ozaki[b,*]

[a] *Policy Board, Bank of Japan, Nihonbashi-Hongokucho 2-1-1, Chuo-ku, Tokyo 103-8660, Japan*
[b] *Faculty of Economics, Keio University, Mita 2-15-45, Minato-ku, Tokyo 108-8345, Japan*

Received 3 September 2003; final version received 23 October 2006
Available online 16 April 2007

Abstract

When firms make a decision about irreversible investment, they may not have complete confidence about their perceived probability measure describing future uncertainty. They may think other probability measures perturbed from the original one are also possible. Such uncertainty, characterized by not a single probability measure but a set of probability measures, is called "Knightian uncertainty." The effect of Knightian uncertainty on the value of irreversible investment opportunity is shown to be drastically different from that of traditional uncertainty in the form of risk. Specifically, an increase in Knightian uncertainty decreases the value of investment opportunity while an increase in risk increases it.

JEL classification: C61; D81; D92

Keywords: Knightian uncertainty; Risk; Irreversible investment; Optimal stopping

1. Introduction and summary

The investment decision of any firm typically involves three features. First, future market conditions are uncertain. Second, the cost of investment is sunk and thus investment is irreversible. Third, investment opportunity does not vanish at once and when to invest becomes a critical decision. This irreversibility of investment under uncertainty and resulting optimal

☆ The research reported here was conducted when the first author was a member of the Faculty of Economics of the University of Tokyo before joining the Policy Board of the Bank of Japan. This study was partially supported by a grant from the Economic and Social Research Institute, Cabinet Office, Government of Japan. However, the views expressed here are strictly personal and in no way represent those of the Bank of Japan, the Economic and Social Research Institute, the University of Tokyo, or Keio University.

* Corresponding author. Fax: +81 3 5427 1578.
E-mail addresses: kiyohiko.nishimura@boj.or.jp (K.G. Nishimura), ozaki@econ.keio.ac.jp (H. Ozaki).

0022-0531/$ - see front matter © 2007 Elsevier Inc. All rights reserved.
doi:10.1016/j.jet.2006.10.011

investment timing problem have attracted considerable attention in recent years, especially after McDonald and Siegel [15] successfully applied financial option pricing techniques to this problem and Dixit and Pindyck [4] related option-theoretic results to neoclassical investment theory.

Most irreversible investment studies, however, assume more than that future market conditions are uncertain. In these studies, future uncertainty is characterized by a certain probability measure over states of nature. This amounts to assuming that the firm is *perfectly certain* that future market conditions are governed by this particular probability measure. However, this assumption may be farfetched: the firm may not be so sure about future uncertainty. It may think other probability measures are also likely and have no idea of the relative "plausibility" of these measures. Uncertainty that is *not* reducible to a single probability measure and thus characterized by a *set* of probability measures is often called *Knightian uncertainty* (see [14,12,13]), or ambiguity in some cases. In contrast, uncertainty that *is* reducible to a single probability measure with known parameters is referred to as *risk* . That is, a firm may face Knightian uncertainty in contemplating investment, facing not a single probability measure but a set of probability measures.

The purpose of this paper is to show that the effect of uncertainty on the value of irreversible investment opportunity differs drastically between risk and Knightian uncertainty. Specifically, the standard result that increase in uncertainty increases the value of irreversible investment opportunities is reversed if uncertainty is not risk but Knightian uncertainty. That is, an increase in Knightian uncertainty (properly defined) reduces the value of an irreversible investment opportunity, while the opposite is true for an increase in risk in the form of an increase in variance. In contrast, both of them have the same effect on the value of waiting: they increase the value of waiting and make it more likely.

In this paper, we take a patent as an example of irreversible investment. To highlight the effect of Knightian uncertainty, the firm is assumed to be risk-neutral but *uncertainty-averse* in the sense that it computes the expected profit by using the "worst" element in the set of the probability measures characterizing Knightian uncertainty and chooses its strategy to maximize it (maximin criterion). [1]

Following the standard procedure of irreversible investment studies, we assume that (1) to utilize a patent, the firm has to build a factory and construction costs are sunk after its completion, and (2) the profit flow after the construction is characterized by a geometric Brownian motion with a drift. Then, the firm first calculates the value of the utilized patent, and then contemplates when to build a factory by taking into account the value of the utilized patent and the cost of investment. The firm's problem is thus formulated as an optimal stopping problem in continuous time. [2]

Unlike the standard case, however, we assume that the firm is not perfectly certain that the profit flow is generated by a particular geometric Brownian motion with say, variance σ^2 and drift μ, or equivalently, by a probability measure underlying this geometric Brownian motion,

[1] For axiomatization of such behavior, see Gilboa and Schmeidler [9]. Such behavior is also closely related to the one represented by Choquet-expected-utility maximization. See Schmeidler [21, first appeared in 1982 as a working paper] and Gilboa [8].

[2] The standard procedure is to apply financial option pricing techniques to this problem, exploiting the fact that an *un*-utilized patent can be considered a call option whose primal asset is a *utilized* patent that generates a stochastic flow of profits, and whose exercise price is a fixed cost of building a factory to produce patented products. (For example, see [4].) This approach and the optimal stopping approach are two ways of formulating the same problem and produce the same result.