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Optimal consumption/investment policies with undiversifiable income risk and liquidity constraints

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Abstract

This paper examines the continuous time optimal consumption and portfolio choice of an investor having an initial wealth endowment and an uncertain stream of income from non-traded assets. The income stream is not spanned by traded assets and the investor is not allowed to borrow against future income, so the financial market is incomplete. We solve the corresponding stochastic control problem numerically with the Markov chain approximation method, prove convergence of the method, and study the optimal policies. In particular, we find that the implicit value the agent attaches to an uncertain income stream typically is much smaller in this incomplete market than it is in the otherwise identical complete market. Our results suggest that this is mainly due to the presence of liquidity constraints.

JEL classification: C61; G11

Keywords: Optimal consumption and portfolio policies; Undiversifiable income risk; Liquidity constraints; Wealth equivalent of income; Markov chain approximation

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1. Introduction

The consumption/investment choice of a price-taking investor is a classical problem of financial economics. In two pioneering papers, Merton (1969, 1971) introduced stochastic control techniques to analyze the continuous-time version of the problem for an investor with an additively time-separable utility function. In particular, Merton studied the case where the investor has access to a complete financial market in which risky asset prices follow geometric Brownian motions and the investor's utility function for consumption is of the constant relative risk aversion type. He was able to solve analytically the Hamilton–Jacobi–Bellman (HJB) equation associated with the problem and hence to obtain closed-form expressions for the optimal control policies in feedback form, both for a finite horizon and an infinite horizon.

One of many interesting generalizations of Merton's setting appears when the investor besides having an initial endowment of wealth also receives a stream of income throughout her planning horizon. Merton (1971, Section 7) stated that the optimal policies when the agent has a deterministic stream of income are as if the agent has no income stream but instead adds the capitalized lifetime income flow discounted at the risk-free rate to her initial wealth. However, it is easy to show, see e.g. He and Pagès (1993, Example 1), that under the policies derived this way the wealth process may go below zero. Due to moral hazard and adverse selection problems, it may be impossible for the investor to borrow against future income, so that the investor can only choose her consumption/investment policy among those that keep her financial wealth non-negative.

He and Pagès (1993) study a model where the income rate is spanned, so that the only source of incompleteness is that liquid wealth has to stay non-negative. Using the martingale techniques of Cox and Huang (1989) they find that the presence of liquidity constraints has a smoothing effect on the optimal consumption across time. If the investor expects her income to rise, she will increase her consumption at a smaller rate than if she was not subjected to liquidity constraints. In a similar set-up, El Karoui and Jeanblanc-Picqué (1996) demonstrate that the optimal trading strategy is to invest part of the wealth in the strategy which is optimal in the corresponding unconstrained case, and the remainder in an American put option written on the optimal wealth process in the unconstrained case.[1] They also derive a formula linking the optimal consumption rate to current wealth and current income, and they show that for zero wealth the optimal consumption rate is a smaller fraction of current income in the liquidity constrained case than in the unconstrained case.

[1] Cox and Huang (1989) demonstrated a similar result for the no-income case.