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Neglected chaos in international stock markets: Bayesian analysis of the joint return–volatility dynamical system

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HIGHLIGHTS

- We use a novel Bayesian inference procedure for the Lyapunov exponent.
- We apply the new technique to daily stock data for a group of six countries.
- The interaction between returns and volatility is taken into consideration jointly.
- We employ Sequential Monte Carlo techniques.
- The evidence shows that there is noisy chaos both before and after the crisis.

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ABSTRACT

We use a novel Bayesian inference procedure for the Lyapunov exponent in the dynamical system of returns and their unobserved volatility. In the dynamical system, computation of largest Lyapunov exponent by traditional methods is impossible as the stochastic nature has to be taken explicitly into account due to unobserved volatility. We apply the new techniques to daily stock return data for a group of six countries, namely USA, UK, Switzerland, Netherlands, Germany and France, from 2003 to 2014, by means of Sequential Monte Carlo for Bayesian inference. The evidence points to the direction that there is indeed noisy chaos both before and after the recent financial crisis. However, when a much simpler model is examined where the interaction between returns and volatility is not taken into consideration jointly, the hypothesis of chaotic dynamics does not receive much support by the data ("neglected chaos").

1. Introduction

Nonlinear models have become popular over the last decades because datasets often seem to exhibit nonlinearities. However, very often significant misspecifications result from ignoring irregular periodicity behaviours and strong dependence upon initial conditions. In this context, it would seem meaningful to test for the presence of chaotic behaviours that have not been already detected yet, in other words to test for "neglected chaos". This would make sure that no functional relationships expressing chaotic behaviours are neglected and that no terms with explanatory power have been left out of the model. Such models, from now on, will be referred to as "neglected chaos" approaches/tests. Meanwhile, an artificial neural network

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(ANN) could be used to test for neglected chaos, using its universal approximation ability [1], where – technically speaking – the model becomes an augmented feed-forward q-perceptron ANN.

The field of chaos was developed several decades ago in Physics to explain some strange-looking behaviours that lacked order. Practically, it refers to the study of unpredictable behaviours in systems that are inherently complex. According to Lahmiri [2]: "a chaotic system is a random-looking nonlinear deterministic process with irregular periodicity and sensitivity to initial conditions". Up until recently, the various tests for chaotic behaviour have been implemented primarily in the fields of Science and Engineering, especially in meteorological and climate change data where noise is usually absent. However, over the last years, chaos has been the subject of research in various fields, including those of economics and finance, where a typical approach in these fields has been that the dominant dynamics are of a stochastic nature, usually described by a given probability function. Relatively recently, some researchers in the field of finance and financial economics, have found some evidence in favour of chaotic dynamics. [3] found evidence of chaotic dynamics in the Indian stock market, [4] in the Tehran stock market, [5] in the French CAC 40 index, [6] in the Turkish Lira–USD exchange rate, and [7] in exchange rates. In this context, tests for chaos using ANNs have gained popularity [8,9].

In financial time series, however, it is not sufficient to account for a flexible functional form to represent state dynamics. Time-varying conditional variance is a key characteristic of these series and, often, this is ignored or modelled with simple models such as the EGARCH [2]. Nevertheless, stochastic volatility models are better suited for financial data but considerably more computationally intensive. In this framework, econophysics tools have become available and are nowadays used for modelling complex financial systems. For the advantages of these approaches and for relevant applications see, among others, [10–13]. In the words of BenSaïda [8]: "[F]ew studies have considered studying the dynamics of financial and economic time series in times of political or economic instability to better understand their behaviour from an econophysics perspective".

In this paper, we consider a bivariate dynamical system consisting of returns and their volatility. Both functional forms are modelled via ANNs and feedback is allowed between returns and volatility. We propose a novel way of computing the largest Lyapunov exponent as the dynamical system is inherently stochastic due to the presence of stochastic volatility. More precisely, in this work, we test for the presence of noisy chaotic dynamics before and after the 2008 international financial crisis that could be seen as an Early Warning Mechanism (EWM). EWMs are important elements of a carefully designed macroprudential policy that could potentially help reduce the high risk associated with financial and economic crises. In this context, EWMs should not only have good statistical properties and good forecasting performance but should also be based on testing for the existence of chaos. In this work, the importance of testing for chaos lies on the appropriate technique suggested which is based on a bivariate dynamical system of returns and their volatility, which is a crucial requirement for the EWM.

The data employed consist of stock indices for six major countries, namely: USA, UK, Switzerland, Netherlands, Germany and France, and their corresponding implied volatility indices. The sample covers the period from 15th May 2003 to 25th November 2014 in order to capture the financial crisis.

2. Model

2.1. General

Suppose a time series $\{x_t; t = 1, \ldots, T\}$ has the representation

$$y_t = f(y_{t-L}, y_{t-2L}, \ldots, y_{t-mL}) + u_t,\ \ u_t|\sigma_t \sim N(0, \sigma_t^2), t = 1, \ldots, T, \tag{1}$$

where σ_t^2 is the conditional variance, m is the embedding dimension (or the length of past dependence) and L is the time delay. The state space representation is:

$$F : \begin{bmatrix} y_{t-L} \\ y_{t-2L} \\ \vdots \\ y_{t-mL} \end{bmatrix} \to \begin{bmatrix} y_t = f(y_{t-L}, y_{t-2L}, \ldots, y_{t-mL}) + \varepsilon_t \\ y_{t-L} \\ \vdots \\ y_{t-(m-1)L} \end{bmatrix}. \tag{2}$$

Given initial conditions y_0 and a perturbation $\triangle y_0$ the time series after t periods changes by $\triangle y(y_0, t)$. The Lyapunov exponent is defined as:

$$\lambda = \lim_{\tau\to\infty} \tau^{-1} \ln \frac{|\triangle y(y_0, \tau)|}{|\Delta y_0|}, \tag{3}$$

and measures the average exponential divergence (positive exponent) or convergence (negative exponent) rate between nearby trajectories within time horizons that differ in terms of initial conditions by an infinitesimal amount. The Jacobian matrix J at a point $\boldsymbol{\chi}$ is

$$J^t(\boldsymbol{x}) = \frac{df^t(\boldsymbol{\chi})}{d\boldsymbol{\chi}}. \tag{4}$$