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# On Robustness of Simultaneous Long-Short Stock Trading Control with Time-Varying Price Dynamics

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**Abstract:** This paper provides a time-varying extension of the so-called Robust Positive Expectation Theorem which arises in the context of Simultaneous Long-Short (SLS) stock trading. In the literature, the original version of the theorem applies to an idealized market with prices governed by a constant-parameter Geometric Brownian Motion (GBM). By this we mean that the process drift $\mu$ and voltility $\sigma$ are time-invariant. In this context, the SLS strategy leads to the guarantee of a positive expected trading gain at every instant of time. In this paper, this result is generalized to GBM price dynamics with these two parameters allowed to be continuously time-varying with no restrictions on their rate of change. The motivation for considering this more general class of stock-price variations is the following: At first glance, it appears that these time-varying price dynamics are "unbeatable" in the sense that there cannot be a stock-trading strategy which still leads to robust positive expectation. In this regard, the "surprising" result in this paper is that the same SLS controller used in the time-invariant case also works in the time-varying case. These results strengthen the case made in earlier literature that linear feedback control in stock trading is worthy of further research.

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## 1. INTRODUCTION

A new line of research has emerged from the control community which explores the efficacy of classical feedback control strategies in a stock trading context; see [1]-[4] and their bibliographies for an extensive review of this literature. This research begins from the point of view that stock trading can be represented as a closed-loop feedback system, as shown in Figure 1; see [4] for a detailed description of this new paradigm. In this figure, the feedback controller is contained in the trader block, which receives signals from the broker such as stock prices, current account value, gains or losses, and other information. In turn, the trader decides on a dollar investment level, $I(t)$, in the stock.

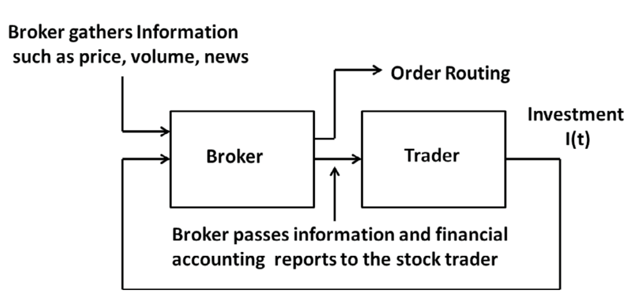


Fig 1: Feedback Loop Involving Trader and Broker

When the control algorithm used by the trader to generate $I(t)$ does not depend on an explicit model for prices, it is referred to as "model-free." For example, a static feedback mapping from the trader's cumulative gains and losses $g(t)$ to the investment level $I(t) = f(g(t))$ illustrates this idea. More generally, results along these model-free lines were first obtained in [5] and [6]. One of the goals of this line of work has been to establish robustness theorems that certify properties of the closed-loop system with respect to classes of stock price dynamics. Subsequently, for such a trading scheme, the guarantee of a positive expected profit over a class of stock price paths is termed the *Robust Positive Expectation* (RPE) property.

Given the setting described above, the focal point here is the so-called Simultaneous Long-Short (SLS) feedback strategy which first appeared in [1] for the class of continuously differentiable stock price paths. Such a strategy involves a pair of linear feedback controllers, one long and one short, which are used to generate the investment level. The actual number of dollars in the trade is obtained by "netting out" these two components; see Section 3.

More recently, various extensions of SLS results have been proved. For example, in [4], the RPE property for the SLS strategy was established for the class of Geometric Brownian Motion price paths with unknown but constant drift and volatility parameters. In [7], it was shown that PI based feedback trading strategies also possess the RPE property over the set of constant parameter GBM price paths. More recently, results in [8] extend the RPE property for the SLS strategy to include price paths with discontinuous jumps. A further extension is provided in [9] where a discrete-time version with delay is considered.

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The purpose of this paper is to provide a generalization of the Robust Positive Expectation Theorem to the set of Geometric Brownian Motion stock price dynamics with *unknown* and *time-varying* parameters. Namely, in contrast to existing literature, the drift $\mu(t)$ and volatility $\sigma(t)$ are not assumed to be constant. Moreover, since neither the sign nor rate of change of the drift $\mu(t)$ is restricted, the history of the price up to time $t$ provides no predictive power for $t' > t$. The existence of such a time-varying extension is noteworthy since this is a class of price dynamics that appears to be "unbeatable."

## 2. PRICE DYNAMICS AND OUR CLAIMS

In this section, we begin the technical exposition with a mathematical description of the assumed price dynamics, followed by our specific claims. Indeed, we now consider a frictionless idealized market characterized by perfect liquidity, continuous trading, no transaction costs, and price taking. That is, traders are assumed to be able to trade any number of shares without affecting the current price and with no commission costs.

The stock price $p(t)$ is governed by the time-varying Geometric Brownian Motion (GBM)

$$\frac{dp}{p} = \mu(t)dt + \sigma(t)dZ_t$$

where $Z_t$ is a standard Wiener process, and the *drift* $\mu(t)$ and *volatility* $\sigma(t)$ are assumed to be continuous functions of time. We include no restriction on the rates of variation of $\mu(t)$ and $\sigma(t)$, and it is also assumed that the trader can borrow and lend at the risk-free rate of interest, which, without loss of generality, we assume to be zero.[1] Note that the assumptions being used here also pervade many of the fundamental papers on markets; e.g., see [11] and [12].

Letting $I(t)$ be the instantaneous dollar investment, the resulting stochastic equation for the cumulative trading gain or loss $g(t)$ is given by

$$dg = \frac{dp}{p}I(t) = (\mu(t)dt + \sigma(t)dZ_t)I(t).$$

### *2.1 Overview of Main Result*

With the simple notation above and no further assumptions, the main result of this paper is easy to paraphrase: There exists a trading rule, namely the SLS algorithm for varying the investment function $I(t)$, which is robust to arbitrary continuous variations in $\mu(t)$ and $\sigma(t)$ and guarantees an "edge." More precisely, for continuously time-varying $\mu(t)$ with unknown sign, magnitude, and non-vanishing integral, and for continuously time-varying $\sigma(t) > 0$, we prove positivity of the expected gain-loss function $g(t)$. Namely,

$$\mathbb{E}[g(t)] > 0$$

for all $t > 0$. That is, regardless of the parameters governing the GBM, the trader robustly expects to make money on every trade. It is in this sense that the positive expectation value above is said to be "robust."

[1] A non-zero risk-free rate can be easily accommodated by choosing the risk-free asset as numeraire. In this case, the theory to be described in the sequel can be extended to show that the trader is guaranteed an expected return in excess of the risk-free rate.

### *2.2 Lack of Predictability*

Given the admissibility of arbitrarily fast time-variations in the drift $\mu(t)$ and volatility $\sigma(t)$ above, some obvious strategies which would work well in the time-invariant case are ruled out. That is, when $\mu$ and $\sigma$ are constant, one obvious method to gain an "edge" is to estimate the sign of the drift $\mu$ over some training period and then go long if $\mu > 0$ and short if $\mu < 0$. Surprising as it may seem, the robust positive expectation for $g(t)$ is assured in the absence of predictability of $\mu(t)$ and $\sigma(t)$. Our scenario is such that there is no ability to take advantage of slow variations to predict the price $p(t)$. Only reaction is possible.

### *2.3 Embellishments of Main Result*

In addition to the positive expected value result, this paper also includes some additional embellishments. That is, we provide details about the probability distribution of the trading gain-loss function $g(t)$. This includes a closed-form solution for its variance, a description of its density function and a complete characterization of the leverage requirements of the SLS strategy.

## 3. THE SLS TRADING RULE

As in the work of [1], [2] and [4], we take the instantaneous dollar investment $I(t)$ to be the sum of two positions, a long trade $I_L(t) > 0$ and a short trade $I_S(t) < 0$. That is,

$$I(t) \doteq I_L(t) + I_S(t).$$

To facilitate understanding of the trading strategy, it is convenient to temporarily view $I_L$ and $I_S$ as two separate trades which can be "netted out" when implemented in a brokerage account; e.g., at some time $t > 0$, if $I_L = \$7,000$ and $I_S = -\$2,500$, then $I(t) = \$4,500$ is the net position which is long. For pedagogical purposes, it is convenient to view these two trades as running simultaneously.

Now, to complete the description of the trading strategy, we need to specify the rules for modification of $I_L(t)$ and $I_S(t)$ over time. Indeed, letting $g_L(t)$ and $g_S(t)$ denote the cumulative trading gain or loss over $[0, t]$ from the long and short trades respectively, taking $I_0 > 0$ to be the initial investment at time $t = 0$ and $K > 0$, the so-called feedback parameter, the long and short investments are given by the linear feedbacks

$$I_L(t) \doteq I_0 + Kg_L(t) \quad \text{and} \quad I_S(t) \doteq -I_0 - Kg_S(t)$$

with resulting net investment

$$I(t) = I_L(t) + I_S(t) = K(g_L(t) - g_S(t)).$$

Finally, since there are no gains or losses at time zero, the condition $g_L(0) = g_S(0) = 0$ makes it apparent that $I_0 > 0$ corresponds to the initial investment in each trade, and $K > 0$, the feedback parameter, while arbitrary in our analysis, is a measure of how "aggressively" the trade is incremented or decremented over time. To summarize, the block diagram in Figure 1 shows all the interconnections of feedback variables which describe the SLS trading scheme.

### *3.1 Adaptation Without a Price Model*

To complete the explanation of SLS trading, we make a number of simple observations: Since the long and short

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