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Symbolic complexity of volatility duration and volatility difference component on voter financial dynamics

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ABSTRACT

The nonlinear complexity of volatility duration and volatility difference component based on voter financial dynamics is investigated in this paper. The statistic – volatility difference component is first introduced in this work, in an attempt to study the volatility behaviors comprehensively. The maximum change rate series and the average change rate series (both derived from the volatility difference components) are employed to characterize the volatility duration properties of financial markets. Further, for the proposed series model and the proposed financial statistic series (which are transformed to symbolic sequences), the permutation Lempel–Ziv complexity, a novel complexity measure, is introduced to study the corresponding randomness and complexity behaviors. Besides, Zipf analysis is also applied to investigate the corresponding Zipf distributions of the proposed series. The empirical study shows the similar complexity behaviors of volatility between the proposed price model and the real stock markets, which exhibits that the proposed model is feasible to some extent.

1. Introduction

Recent researches on the financial field demonstrate that the financial market is a complex and dynamical system whose fluctuations often represent strong nonlinear and dynamical characteristics, and the interactions of financial market participants have attracted many financial researchers' attentions. Over the past ten years, many new interacting particle models have been proposed to study the financial markets [1–11], such as Bertrand and Cournot competitions in continuous time [1], reduced-form point process model [2], correlated default model [3] and so on. Many financial behaviors, including large pools of loan [4], portfolio losses [5–8], inter-bank lending and borrowing [9–11], are studied with these models. The stock market is an important part of financial markets, where there are some common properties called stock market stylized empirical facts, including fat tails, absence of autocorrelation, volatility clustering and so on [12]. In addition, with the governments' deregulation of stock markets all over the world, it is becoming a vital topic to capture the dynamics of the forward prices of stock markets in risk management, derivatives pricing and physical assets valuation. The modeling of stock markets, aiming at understanding price fluctuation dynamics, demands to establish a mechanism for the formation of the stock price. In the past years, considering the similarity between stock markets and physical systems, some scholars apply the statistical physics theories and methods to perform the empirical research on the stock markets [13–21]. Some agent-based interacting models from the percolation networks, the Potts dynamic system, etc. [13,14,18–21], have been established attempting to reproduce the complex and dynamical behaviors of stock markets. For example, Stauffer and Penna [14] developed a price model by the lattice percolation system and exhibited the existence of the fat tails for the return process; Hong and Wang [19] modeled the stock dynamics by the Potts model and explored the correlation of the logarithmic returns. The voter model, one of discrete agent-based models of opinion dynamics, is a stochastic interacting Markov process [20]. The voter process represents a voter's attitude affected by his neighbors' opinions at times distributed on a particular topic according to a stochastic rule [22–26]. Taking into account most of agents in the stock market trade stocks basing on their opinions to the investment information, we suppose that the interaction among the stock market agents is random, then utilize the voter interaction system to model the dynamics of the agents' opinions attempting to reproduce financial price fluctuations and volatility behaviors. Then we investigate the nonlinear phenomena of volatilities of the voter price model.

It is very important to understand the volatility behavior of financial markets, since it helps investors quantify the risk, optimize the portfolio and so on. The absolute returns, which is also called volatility series, is the key target for financial volatility be-

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havior research, and many new methods of volatility analysis are developed [27–29] for better understanding volatility behaviors, for example, the return interval analysis, characterizing the occurrence of volatilities within a certain range, are proposed in Refs. [27,28]. In this paper, we introduce a new statistic – volatility difference component (VDC) to characterize the volatility behaviors of the financial model in the volatility duration length, and develop a new method to explore the nonlinear phenomena of volatilities for the proposed price model, by transferring the volatility series to three kinds of volatility duration statistical series, the volatility duration series, the maximum change rate series and the average change rate series. The volatility duration series reflects local rising or falling volatility duration length, while the rest two series (which are derived from the volatility difference components) record the maximum change rate and the average change rate of volatility difference components in every volatility duration length. The exploration on these three series is useful for further understanding the volatility behaviors of financial markets and the proposed price model. In the empirical research, the randomness and complexity of the proposed volatility duration statistical series are investigated by introducing a novel approach – permutation Lempel–Ziv complexity (PLZC). The dynamic behaviors of volatilities are also studied by the Zipf analysis with different parameters of thresholds and timescales. The daily closing price data of Shanghai Stock Exchange (SSE) Composite Index and Shenzhen Stock Exchange (SZSE) Component Index are selected as the real empirical data for comparison.

2. Voter interacting financial price model

2.1. Voter interaction system

The stochastic voter interacting particle system was introduced independently by Clifford and Sudbury [22] and Holley and Liggett [23]. The evolution mechanism of the voter system starts with voters located at the nodes of lattice $\mathbb{Z}^d$, which might have one of two possible opinions on a political issue (in favor “1” or against “0”) at independent exponential times. A voter reassesses his opinion by choosing a neighbor at random with certain probabilities and adopting his position. Let ξ_τ be the set of voters in favor, which is a continuous time Markov process. The dynamics of the process is specified by the collection of transition rates $c(x,\xi)$ [24–26]. For any $\xi \in \{0,1\}^{\mathbb{Z}^d}$, the state of $x \in \mathbb{Z}^d$ flips following the transition rates

$$0 \to 1 \text{ at rate } \lambda \sum_{y\in\mathbb{Z}^d} p(x,y)\, \mathbb{I}_{\{\xi(y)=1\}} \tag{1}$$

$$1 \to 0 \text{ at rate } \sum_{y\in\mathbb{Z}^d} p(x,y)\, \mathbb{I}_{\{\xi(y)=0\}} \tag{2}$$

where $\mathbb{I}$ is the indicator function, $p(x,y) \geq 0$ for $x, y \in \mathbb{Z}^d$, and $\sum_{y\in\mathbb{Z}^d} p(x,y) = 1$ for all $x \in \mathbb{Z}^d$. The transition probability $p(x,y)$ is translation invariant and symmetric, and the voter process with those transition probabilities is irreducible. If a node $x \in \mathbb{Z}^d$ is occupied by 1 (respectively, 0), then, at rate 1 (respectively, λ), it picks a node $y \in \mathbb{Z}^d$ with probability $p(x,y)$, then adopts the state of the voter at y. The stochastic dynamics of voter model ξ_τ on a configuration space $\{0,1\}^{\mathbb{Z}^d}$ is given as the form of generator by

$$\mathcal{A}g(\xi) = \sum_{x\in\mathbb{Z}^d} c(x,\xi)[g(\xi^x) - g(\xi)] \tag{3}$$

where the function g on $\{0,1\}^{\mathbb{Z}^d}$ depends on the finitely many coordinates, and $\xi^x(z) = \xi(z)$ if $z \neq x$, $\xi^x(z) = 1 - \xi(x)$ if $z = x$, for $x, z \in \mathbb{Z}^d$. In details: (i) if $x \in \xi_\tau$, then x becomes vacant at a rate equal to the number of vacant neighbors; (ii) if $x \notin \xi_\tau$, then x becomes occupied at a rate equal to λ times the number of occupied neighbors, where λ is an intensity, which is called the carcinogenic advantage in the voter process. When $\lambda = 1$, the model is called the voter model, and for $\lambda > 1$ it is called the biased voter model. Let $\xi_\tau^{\{A\}}$ denote the state at time τ with the initial state set $\xi_0^{\{A\}} = \{A\}$, and let $\xi_\tau^{\{0\}}(x)$ be the state of $x \in \mathbb{Z}^d$ at time τ with the initial state $\xi_0^{\{0\}} = \{0\}$, which means that only the original point $\{0\}$ of $\mathbb{Z}^d$ is occupied in the initial state (at $\tau = 0$) of the process $\xi_\tau^{\{0\}}$. More generally, the initial distribution is considered as υ_ρ, the product measure with density ρ (each node is independently occupied by probability ρ), and let $\xi_\tau^{\upsilon_\rho}$ be the voter model with initial distribution υ_ρ.

For the biased voter model ($\lambda > 1$), there is a “critical value” for the process on $\Omega = \{0,1\}^{\mathbb{Z}^d}$, the critical value λ_c is defined as [13,14]

$$\lambda_c = \inf\{\lambda : P(|\xi_\tau^{\{0\}}| > 0,\ \text{for all } \tau \geq 0) > 0\} \tag{4}$$

where $|\xi_\tau^{\{0\}}|$ is the cardinality of $\xi_\tau^{\{0\}}$. Suppose $\lambda > \lambda_c$, then there is convex set C so that on $\Omega_\infty = \{\xi_\tau^{\{0\}} \neq \emptyset,\ \text{for all } \tau\}$, for any $\epsilon > 0$ and for all τ sufficiently large

$$(1-\epsilon)\tau C \cap \mathbb{Z}^d \subset \xi_\tau^{\{0\}} \subset (1+\epsilon)\tau C \cap \mathbb{Z}^d. \tag{5}$$

If $\lambda \leq \lambda_c$, for some positive $\gamma(\lambda)$, then

$$P(\xi_\tau^{\{0\}} \neq \emptyset) \leq e^{-\gamma(\lambda)\tau}. \tag{6}$$

The above theory shows that, on d-dimensional lattice, the process becomes vacant exponentially for $\lambda < \lambda_c$, and survives with positive probability for $\lambda > \lambda_c$.

2.2. Construction of financial price model

The financial price dynamics based on the voter process is formulated as follows. Suppose that the investment information leads to the fluctuation of a stock price, and there are three kinds of information including buying, selling and neutral, which classify the investors into their corresponding groups. Assume that each trader can trade the stock several times at each day $t \in \{1,2,\cdots,N\}$, but at most, one unit number of the stock at each time. Let l be the time length of one trading day, we denote the stock price at time τ in the tth trading day by $P_t(\tau)$, where $\tau \in [0,l]$. Suppose that the stock market is made up of $2m+1$ (m is large enough) invertors, who are located in a line $\{-m,\cdots,-1,0,1,\cdots,m\} \subset \mathbb{Z}$ (similarly for a d-dimensional lattice $\mathbb{Z}^d$). At the starting of each trading day, only the investor at the origin site “0” receives some information. And a random variable ζ_t with values 1, −1, 0 represents that this investor holds buying opinion, selling opinion or neutral opinion with probabilities p_1, p_{-1} or $1-p_1-p_{-1}$ respectively. Then this investor sends bullish, bearish or neutral signal to his nearest neighbors. According to the voter dynamical system, investors can affect each other or the information can be disseminated, which is considered as the main factor of price fluctuations for the stock market.

For a trading day $t \leq N$ and $\tau \in [0,l]$, let

$$B_t(\tau) = \zeta_t \times \frac{|\xi_\tau^{\{0\}}|}{2m+1}, \quad \tau \in [0,l] \tag{7}$$

where $|\xi_\tau^{\{0\}}| = \sum_{w=-m}^{m} \xi_\tau^{\{0\}}(w)$. The stock price process at tth trading day is given as [30,31]

$$P_t(\tau) = e^{\alpha_t B_t(\tau)} P_{t-1}(\tau), \quad \tau \in [0,l] \tag{8}$$

$$P_t(\tau) = P_0 e^{\sum_{i=1}^{t} \alpha_i B_i(\tau)}, \quad \tau \in [0,l] \tag{9}$$