Insurance: Mathematics and Economics 76 (2017) 149–163

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Unit-linked life insurance policies: Optimal hedging in partially observable market models

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ARTICLE INFO

Article history:
Received December 2016
Received in revised form July 2017
Accepted 14 July 2017
Available online 7 August 2017

JEL classification:
C02
G11
G22

MSC 2010:
91B30
60G35
60G40
60J60

Keywords:
Unit-linked life insurance contract
Progressive enlargement of filtration
Partial Information
Local risk-minimization
Föllmer–Schweizer decomposition
Markov processes

ABSTRACT

In this paper we investigate the hedging problem of a unit-linked life insurance contract via the local risk-minimization approach, when the insurer has a restricted information on the market. In particular, we consider an endowment insurance contract, that is a combination of a term insurance policy and a pure endowment, whose final value depends on the trend of a stock market where the premia the policyholder pays are invested. To allow for mutual dependence between the financial and the insurance markets, we use the progressive enlargement of filtration approach. We assume that the stock price process dynamics depends on an exogenous unobservable stochastic factor that also influences the mortality rate of the policyholder. We characterize the optimal hedging strategy in terms of the integrand in the Galtchouk–Kunita–Watanabe decomposition of the insurance claim with respect to the minimal martingale measure and the available information flow. We provide an explicit formula by means of predictable projection of the corresponding hedging strategy under full information with respect to the natural filtration of the risky asset price and the minimal martingale measure. Finally, we discuss applications in a Markovian setting via filtering.

1. Introduction

Unit-linked life insurance contracts are life insurance policies whose benefits depend on the performance of a certain stock or a portfolio traded in the financial market. For the last years these contracts have experienced a clamorous success, driven by low interest rates, which have considerably reduced the returns of the classic management, and the new Solvency II rules on the insurance regulatory capital, which made the unit-linked much more affordable for the companies, in terms of lower absorption of capital. According to Gantenbein and Mata (2008, Chapter 10), a *unit-linked life insurance policy* is "basically a mixed life insurance that combines term coverage with a saving and an investment component". Unlike the traditional mixed life insurance, in these contracts premia are invested by the insurance company in the financial market on behalf of the policyholder who decides how to invest the capital. Among these products, we may distinguish at least three different kinds of policies based on the payoff structure:

- *pure endowment* contract that promises to pay an agreed amount if the policyholder is still alive on a specified future date;
- *term insurance* contract that pays the benefit if the policy-holder dies before the policy term;
- *endowment insurance* contract which is a combination of the above contracts and guarantees that benefits will be paid by the insurance company, either at the policy term or after the insured death.

The goal of this paper is to find an optimal hedging strategy for a given endowment insurance contract in a general intensity-based model where the mortality intensity, as well as the drift in the risky asset price dynamics affecting the benefits for the policyholder, is *not observable* by the insurance company, and *mutual dependence*

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http://dx.doi.org/10.1016/j.insmatheco.2017.07.005
0167-6687/

between the stock price trend and the insurance portfolio is allowed. To the best of our knowledge, this is the first time that the problem of hedging a unit-linked life insurance policy is studied under partial information, without assuming independence between the financial and the insurance markets.

Precisely, we propose a suitable combined financial-insurance market model, where the financial market consists of a riskless asset, whose discounted price is equal to 1, and a risky asset, with discounted price process denoted by S. The price process S is represented by a geometric diffusion, whose drift depends on an exogenous unobservable stochastic factor X, correlated with S. The insurance company issues an endowment insurance contract with maturity of T years for an individual whose remaining lifetime is represented by a random time τ.

Modeling the death time of an individual is a fundamental issue to be addressed when dealing with insurance problems. Here, we propose a modeling framework for life insurance liabilities that is also well suited to describe defaultable claims, as the time of death can be handled in a similar manner to the default time of a firm. Then, we take the analogies between mortality and credit risk into account and follow the intensity-based approach of reduced-form methodology, see e.g. Bielecki and Rutkowski (2004) and references therein. The death time τ is described by a nonnegative random variable, which is not necessarily a stopping time with respect to the initial filtration $\mathbb{F}$ generated by the underlying Brownian motions driving the dynamics of the pair (S, X). As mentioned above, we do not assume independence between the random time of death and the financial market, and characterize our setting via the progressive enlargement of filtration approach, see, e.g., the seminal works by Jeulin and Yor (1978, 1985) and Jeulin (1980). This technique is widely applied to reduced-form models for credit risk, as in Bielecki et al. (2004, 2006a, c), Elliott et al. (2000) and Kusuoka (1999). Moreover, applications to insurance problems can be found in Biagini et al. (2017), Barbarin (2007), Choulli et al. (2015) and Li and Szimayer (2011) in a complete information setting.

Here, we consider an enlargement of the filtration $\mathbb{F}$ to make τ a stopping time and we denote it by $\mathbb{G}$. The available information to the insurance company is represented by a subfiltration $\widetilde{\mathbb{G}}$ of $\mathbb{G}$, which contains the natural filtration of S and ensures that τ is still a stopping time. This means that, at any time t, the insurer may observe the risky asset price and knows if the policyholder is still alive or not.

The endowment insurance contract can be treated as a contingent claim in the hybrid market model given by the financial securities and the insurance portfolio, and the objective is to study the hedging problem for the insurance company. Analogously to Bielecki et al. (2006b) and Biagini and Cretarola (2012), we assume that hedging stops after the earlier between the policyholder death τ and the maturity T: this allows to work with stopped price processes and guarantees that the stopped Brownian motions, that drive the financial market, are also Brownian motions with respect to the enlarged filtration. As a consequence, we do not need to assume the *martingale invariance property*, also known as *H-hypothesis*, see e.g. Bielecki and Rutkowski (2004). Since the underlying market is incomplete due to the mortality risk and the presence of the unobservable stochastic factor X, it is necessary to select one of the techniques for pricing and hedging in incomplete markets. Then, we choose, among the quadratic hedging methods, the *local risk-minimization* approach (see e.g. Schweizer (2001) for further details). The idea of this technique is to find an optimal hedging strategy that perfectly replicates the given contingent claim with minimal cost, within a wide class of admissible strategies that in general might not necessarily be self-financing. Locally risk-minimizing hedging strategies can be characterized via the Föllmer–Schweizer decomposition of the random variable representing the payoff of the given contingent claim, see e.g. Schweizer (1995, 2001) for the full information case and Ceci et al. (2014a, 2015b) under incomplete information. This quadratic hedging approach has been successfully applied to the hedging problem of insurance products, see e.g. Biagini et al. (2017, 2016), Choulli et al. (2015), Dahl and Møller (2006), Møller (1998, 2001) and Vandaele and Vanmaele (2008) for the complete information case and Ceci et al. (2015a) under partial information.

In this paper, we introduce the stopped Föllmer–Schweizer decomposition under partial information and in Proposition 4.10 we characterize the optimal hedging strategy in terms of the integrand in this decomposition. In this sense, we extend Biagini and Cretarola (2012, Proposition 3.7) to the partial information framework. We also introduce the corresponding Galtchouk–Kunita–Watanabe decomposition with respect to the minimal martingale measure. In Theorem 4.16, we provide equivalence of these decompositions and, using the result stated in Proposition 4.15, the relation between the optimal hedging strategy under partial information and that under full information via predictable projections. In the case where the mortality intensity has a Markovian dependence on the unobservable stochastic factor X, we can compute the optimal hedging strategy in a more explicit form by means of filtering problems. Pricing and hedging problems for contingent claims under incomplete information using filtering techniques have been studied in credit risk context, in Frey and Runggaldier (2010), Frey and Schmidt (2012), Tardelli (2015) and in the insurance framework in Ceci et al. (2015a) under the hypothesis of independence between the financial and the insurance markets.

The paper is organized as follows. In Section 2 we introduce the combined financial-insurance market model in a partial information scenario via progressive enlargement of filtrations. The semimartingale decompositions of the stopped risky asset price process with respect to the enlarged filtrations $\mathbb{G}$ and $\widetilde{\mathbb{G}}$ respectively, can be found in Section 3. In Section 4 we provide a closed formula for the locally risk-minimizing hedging strategy under incomplete information for the given endowment insurance contract by means of predictable projections. Finally, in Section 5 we discuss the problem in a Markovian framework, where the mortality intensity depends on the unobservable stochastic factor and apply the filtering approach to compute the optimal hedging strategy. In addition, we address the issue of the hazard process and the martingale hazard process of τ under restricted information in Appendix A. Some technical results on the optional and predictable projections under partial information and certain proofs can be found in Appendix B.

2. The setting

We consider the problem of an insurance company that wishes to hedge a unit-linked life insurance contract. The value of the policy depends on the performance of the underlying stock or portfolio traded on the financial market as well as the remaining lifetime of the policyholder. Therefore, the insurer is exposed to both financial and mortality risks. The nature of the problem suggests to construct a combined financial-insurance market model and treat the life insurance policy as a contingent claim. We will define the suitable modeling framework via the progressive enlargement of filtration approach, which allows for possible dependence between the financial market and the insurance portfolio. As first step, we introduce the underlying financial market model.

2.1. The financial market model

Let $W = \{W_t,\ t \in [0, T]\}$ and $B = \{B_t,\ t \in [0, T]\}$, with $W_0 = B_0 = 0$, be two independent one dimensional Brownian motions on the complete probability space $(\Omega, \mathcal{F}, \mathbf{P})$, with T denoting a fixed