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Stability concepts in matching under distributional constraints

Yuichiro Kamada [a,1], Fuhito Kojima [b,*,1]

[a] *Haas School of Business, University of California, Berkeley, Berkeley, CA 94720, United States*
[b] *Department of Economics, Stanford University, Stanford, CA 94305, United States*

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Abstract

Many real matching markets are subject to distributional constraints. To guide market designers faced with constraints, we propose new stability concepts. A matching is strongly stable if satisfying blocking pairs inevitably violates a constraint. We show that a strongly stable matching may not exist, and that existence is guaranteed if and only if all distributional constraints are trivial. To overcome this difficulty, we propose a more permissive concept, weak stability. We demonstrate a weakly stable matching always exists, implies efficiency, and is characterized by standard normative axioms. These results are obtained in a more general environment than those in existing studies, accommodating a wide variety of applications.

* Corresponding author.
E-mail addresses: y.cam.24@gmail.com (Y. Kamada), fkojima@stanford.edu (F. Kojima).

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1. Introduction

Many real matching markets are subject to distributional constraints. For example, medical residency matching in Japan is subject to the "regional cap" constraint, which is an upper-bound constraint on the total number of residents that can be assigned in each region. Policies that are mathematically equivalent to the regional-cap policy can be found in many different applications, such as graduate school admission in China, college admission in several European countries, residency match in the U.K., and teacher assignment in Scotland.

For cases without any distributional constraint, the theory of two-sided matching has been extensively studied ever since the seminal contribution by Gale and Shapley (1962), and it has been applied to the design of clearinghouses in various markets in practice. Stability has emerged as the key feature to the success of matching market design; a matching is stable if there is no blocking pair, that is, there is no pair of agents (say a doctor and a hospital) who prefer matching with each other to accepting the current matching. Unfortunately, all stable matchings may violate the given distributional constraint. This fact poses a challenge to market designers faced with such constraints.

The present paper addresses this challenge by analyzing stability concepts that respect the existence of distributional constraints. In order to guide our pursuit of the "right" stability concepts, we build upon the idea that there are different types of blocking pairs. Based on this idea, we consider stability concepts that require certain blocking pairs be eliminated while tolerating others.

Depending on the requirement placed on tolerated blocking pairs, we consider two related concepts, strong and weak stability. We begin by defining strong stability. We say that a matching is strongly stable if satisfying a blocking pair inevitably results in a violation of the distributional constraint.

While strong stability is perhaps the most natural stability concept under distributional constraints, we find a number of senses in which this concept is too demanding and is unlikely to be useful for market designers faced with distributional constraints. First, we find that a strongly stable matching does not necessarily exist, unlike a stable matching without constraints. In addition, we show that no mechanism is strategy-proof for doctors and produces a strongly stable matching whenever one exists.

Given these findings, we seek necessary and sufficient conditions under which these negative conclusions about strong stability can be avoided. Our characterization result demonstrates that the cases under which the above negative conclusions can be avoided are exactly the cases in which the distributional constraint reduces to each individual hospital's capacity constraint. Thus, the difficulty with strong stability is an inevitable conclusion as long as there exists a nontrivial constraint.

Motivated by these negative conclusions, we introduce a more permissive concept, which we call weak stability. We say that a matching is weakly stable if it eliminates two most intuitive, and therefore likely most worrisome, types of blocking pairs. More specifically, it requires that no blocking pair exists such that either (i) adding a doctor at the blocking hospital does not violate