Games and Economic Behavior ••• (••••) •••–•••

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Contract design and stability in many-to-many matching ☆

John William Hatfield [a], Scott Duke Kominers [b,c,d,*]

[a] McCombs School of Business, University of Texas at Austin, United States
[b] Society of Fellows, Department of Economics, Center of Mathematical Sciences and Applications, Center for Research on Computation and Society, and Program for Evolutionary Dynamics, Harvard University, Cambridge, MA, United States
[c] Harvard Business School, Boston, MA, United States
[d] National Bureau of Economic Research, Cambridge, MA, United States

ARTICLE INFO

Article history:
Received 7 October 2014
Available online xxxx

JEL classification:
C78
C62
D47
L14

Keywords:
Many-to-many matching
Stability
Substitutes
Contract design

ABSTRACT

We develop a model of many-to-many matching with contracts that subsumes as special cases many-to-many matching markets and buyer–seller markets with heterogeneous and indivisible goods. In our setting, substitutable preferences are sufficient to guarantee the existence of stable outcomes; moreover, in contrast to results for the setting of many-to-one matching with contracts, if any agent's preferences are not substitutable, then the existence of a stable outcome can not be guaranteed.

In many-to-many matching with contracts, a new market design issue arises: The design of the contract language can impact the set of stable outcomes. Bundling contractual primitives encourages substitutability of agents' preferences over contracts and makes stable outcomes more likely to exist; however, bundling also makes the contractual language less expressive. Consequently, in choosing contract language, market designers face a tradeoff between expressiveness and stability.

1. Introduction

We develop a model of many-to-many matching in which agents on two opposing sides of a market negotiate over contractual relationships, possibly signing multiple contracts. This setting models several real-world matching markets, such as the United Kingdom Medical Intern match (see Roth and Sotomayor (1990)), the market used to allocate blood from blood banks to hospitals (see Jaume et al. (2012)), and the market for advertising within mobile applications (see Lee et al. (2014)).

☆ This paper contains parts of two earlier working papers entitled "Contract Design and Stability in Matching Markets" and "Many-to-Many Matching with Contracts." The authors appreciate the helpful comments of Eric Budish, Jeremy Bulow, Peter Cramton, Federico Echenique, Sonia Jaffe, Ravi Jagadeesan, Fuhito Kojima, David Laibson, Paul Milgrom, Alexandru Nichifor, Muriel Niederle, Michael Ostrovsky, David Parkes, Philip Reny, Alvin E. Roth, Tayfun Sönmez, Utku Ünver, Alexander Westkamp, E. Glen Weyl, several referees, the editors (Yan Chen and Thomas Palfrey), seminar audiences at the AEA Meetings, Chicago, Harvard, Northwestern, and Stanford, and especially Drew Fudenberg. Much of this work was completed while Hatfield was at the Stanford University Graduate School of Business and Kominers was at the Becker Friedman Institute at the University of Chicago. Hatfield appreciates the hospitality of Harvard Business School, which hosted him during parts of this research. Additionally, Kominers gratefully acknowledges the support of a National Science Foundation Graduate Research Fellowship, National Science Foundation Grants CCF-1216095 and SES-1459912, the Harvard Milton Fund, a Yahoo! Key Scientific Challenges Program Fellowship, a Terence M. Considine Fellowship in Law and Economics funded by the John M. Olin Center at Harvard Law School, and an AMS–Simons Travel Grant.

* Corresponding author. Address: Arthur Rock Center for Entrepreneurship, Harvard Business School, Soldiers Field, Boston, MA, 02163, United States.
E-mail addresses: john.hatfield@utexas.edu (J.W. Hatfield), kominers@fas.harvard.edu (S.D. Kominers).
URLs: http://www.jwhatfield.com/ (J.W. Hatfield), http://www.scottkom.com/ (S.D. Kominers).

http://dx.doi.org/10.1016/j.geb.2016.01.002
0899-8256/

Please cite this article in press as: Hatfield, J.W., Kominers, S.D. Contract design and stability in many-to-many matching. Games Econ. Behav. (2016), http://dx.doi.org/10.1016/j.geb.2016.01.002

One important special case of our model is matching with couples, in which pairs of individuals may choose to act as a single agent that receives (at most) two assignments (see Klaus and Klijn (2005) and Klaus et al. (2007)).[1] Our model also includes as special cases many-to-one matching with contracts (Kelso and Crawford (1982), Hatfield and Milgrom (2005)), many-to-many matching (Sotomayor (1999, 2004), Echenique and Oviedo (2006), Konishi and Ünver (2006)), many-to-many matching with wages (Roth (1985), Blair (1988)), and buyer–seller markets with heterogeneous and indivisible goods.[2]

We show that stable outcomes are guaranteed to exist in the setting of many-to-many matching with contracts when preferences are *substitutable* in the sense that no contract becomes desirable when some other contract becomes available.[3] Moreover, substitutability is necessary for the existence of stable outcomes in the maximal domain sense: if any one agent has preferences that are not substitutable, then there exist substitutable preferences for the other agents such that no stable outcome exists. Our maximal domain result is particularly surprising because no analogous result holds in the Hatfield and Milgrom (2005) model of many-to-one matching with contracts (see Hatfield and Kojima (2008, 2010), Hatfield and Kominers (2016), and Hatfield, Kominers, and Westkamp (2016)).

We discuss the structure of the set of stable outcomes, noting extensions of standard lattice structure and rural hospitals results. We then show that when hospital preferences satisfy a more stringent condition than substitutability (so-called *strong substitutability*) and doctor preferences are substitutable, stability is equivalent to a more stringent solution concept: *strong stability*.[4]

Modeling many-to-many matching with contracts raises a subtle conceptual issue: Whereas in many-to-one matching with contracts the entire relationship between two agents must be specified by a single contract, this requirement—which Kominers (2012) calls *unitarity*—is not necessary in many-to-many matching with contracts. Non-unitarity is present in many important applications. For instance, in the United Kingdom Medical Intern match, a student must find both a surgical and a medical position, and hospitals typically hire multiple students. In that context, students are assigned two separate contracts by the match, even if they end up at the same hospital; that is, the United Kingdom Medical Intern match as practiced is non-unitary. In principle, however, one could bundle contractual terms for any application to impose unitarity—i.e., to represent every possible relationship between a doctor–hospital pair as a single contractual relationship—from the designer's perspective. One key contribution of our work is to show that such bundling may not be optimal: for some applications, including the United Kingdom Medical Intern match, market designers may not want to require unitarity and would prefer to leave contracts unbundled.[5]

Allowing multiple contracts between a doctor–hospital pair highlights the importance of *contract language* design, where by contract language we mean the set of possible relationships between a doctor and a hospital that can be expressed as part of a contractual outcome. More generally, however, the choice of a particular contract language is crucial in determining the set of stable outcomes. For example, consider a setting with a doctor d and a hospital h. Contracts can specify one or two of the following terms: the doctor works in the morning (m); the doctor works in the afternoon (a). The doctor would most prefer to work in both the morning and the afternoon, but would be willing to work just the afternoon shift; he is unwilling to work only the morning shift. The hospital would hire the doctor for any shift—and for both shifts—but would most prefer that the doctor work only in the morning, and would rather hire the doctor full-time than for just the afternoon. We denote by x^{Γ} the contract with terms $\Gamma \subseteq \{m, a\}$. When morning and afternoon shifts are contracted separately, the doctor's preferences P_d over contracts are given by

$$P_d : \left\{x^{\{m\}}, x^{\{a\}}\right\} \succ \left\{x^{\{a\}}\right\} \succ \varnothing \succ \left\{x^{\{m\}}\right\},$$

while the hospital's preferences P_h are given by

$$P_h : \left\{x^{\{m\}}\right\} \succ \left\{x^{\{m\}}, x^{\{a\}}\right\} \succ \left\{x^{\{a\}}\right\} \succ \varnothing.$$

There is no stable contracting outcome: for the set $\left\{x^{\{m\}}, x^{\{a\}}\right\}$, the hospital will not be willing to sign $x^{\{a\}}$; for the set $\left\{x^{\{a\}}\right\}$, both parties prefer that the doctor work full time; the set $\left\{x^{\{m\}}\right\}$ is not individually rational for the doctor; and finally both

[1] In the United States National Resident Matching Program (NRMP), pairs of doctors may apply as couples, submitting a preference list over pairs of job assignments, and potentially being assigned to two jobs (see Roth and Peranson (1999)).

[2] Our model is substantively different from the only previous model of many-to-many matching with contracts—that of Klaus and Walzl (2009)—as we allow a given doctor and hospital to sign multiple contracts with each other. This distinction is material to our results, as we discuss in Section 3.4.

[3] We show that this substitutability concept has a natural interpretation in terms of utility theory: preferences over contracts are substitutable if and only if they can be represented by a submodular indirect utility function over sets of offered contracts (see Section 2.1).

[4] Unlike in many-to-one matching, the set of core many-to-many matchings does not generally correspond to the set of stable many-to-many matchings (Blair (1988); see also Echenique and Oviedo (2006) and Konishi and Ünver (2006)). This problem is still extant in the more general setting of many-to-many matching with contracts; hence, we follow Echenique and Oviedo (2006) and Klaus and Walzl (2009) in studying a solution concept alternative to and stronger than stability. Our strengthened stability concept, *strong stability*, is stronger than the similar concept of *setwise stability* studied by Echenique and Oviedo (2006) and Klaus and Walzl (2009).

[5] Unitarity is sometimes problematic for technical reasons as well (see Section 3.4). Nevertheless, imposing unitarity can sometimes be beneficial because of the additional structure it adds: As Echenique (2012) showed, the many-to-one matching with contracts model with substitutable preferences embeds into the seemingly simpler matching with salaries (and gross substitutes) framework of Kelso and Crawford (1982); this allows efficient proofs of the main existence and lattice results for matching with contracts, by appeal to the analogous results for the Kelso and Crawford (1982) framework. The Echenique (2012) result carries over to many-to-one matching with contracts settings with unilaterally substitutable preferences (although via a different embedding—see Schlegel (2015)) and to unitary many-to-many matching with contracts models (Kominers (2012)).

Please cite this article in press as: Hatfield, J.W., Kominers, S.D. Contract design and stability in many-to-many matching. Games Econ. Behav. (2016), http://dx.doi.org/10.1016/j.geb.2016.01.002