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Unilateral substitutability implies substitutable completability in many-to-one matching with contracts

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ABSTRACT

We prove that the *unilateral substitutability* property introduced in Hatfield and Kojima (2010) implies the *substitutable completability* property from Hatfield and Kominers (2014). This paper provides a novel linkage between these two sufficient conditions for the existence of a stable matching in many-to-one matching markets with contracts. A substitutable completion of a preference is a *substitutable* preference created by adding some sets of contracts to the original preference order. We provide an algorithm which when operated on the *unilaterally substitutable* preferences produces such a substitutable completion. Thus it provides a constructive proof of the connection between the two properties.

1. Introduction

The literature on many-to-one matching markets with contracts started with the seminal contributions by Kelso and Crawford (1982)[1] and Hatfield and Milgrom (2005).[2,3] The practical applications of these markets with contracts have recently been investigated in some interesting contexts like cadet branching (Sönmez, 2013; Sönmez and Switzer, 2013), matching with regional caps (Kamada and Kojima 2012, 2015a), and diversity design in school choice (Kominers and Sönmez, 2013).

Roth (1990) described the importance of *stability* for practical matching markets. He observed that the markets which generated a *stable* outcome continued to operate over longer periods of time than the ones which did not guarantee this property. For many-to-one matching with contracts, the literature has provided many conditions on the agents' preferences which are sufficient for stability, e.g. *substitutability* (Kelso and Crawford, 1982; Roth, 1984), *unilateral substitutability* (Hatfield and Kojima, 2010), *bilateral substitutability* (Hatfield and Kojima, 2010), and *substitutable completability* (Hatfield and Kominers, 2014). However, the literature has not fully explored the connections between these sufficient conditions, which might be useful for practical applications. This paper shows that unilateral substitutability implies substitutable completability.

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[1] Kelso and Crawford (1982) builds on the analysis of Crawford and Knoer (1981).

[2] This generalized the many-to-one matching market from Gale and Shapley (1962). That has in turn been further extended and generalized to interesting domains, viz. supply chain networks (Ostrovsky, 2008; Hatfield and Kominers, 2012) and many-to-many matching markets with contracts (Hatfield and Kominers, 2013).

[3] Echenique (2012) has shown the surprising isomorphism between Kelso and Crawford (1982) and Hatfield and Milgrom (2005).

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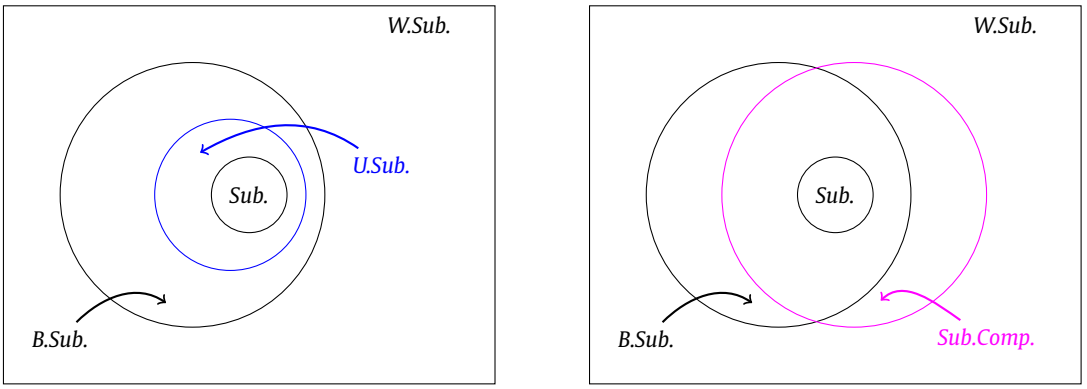
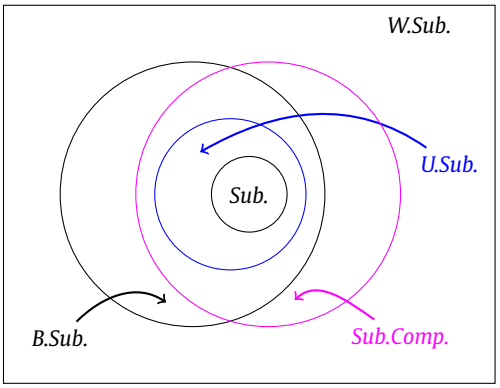
The preference of an agent on the many-side of the market satisfies the *substitutability* condition when the agent does not have any complementarities between contracts.[4] In other words, the agent views each contract independently and never finds a contract that is rejected from some set of contracts to be acceptable only in the presence of another contract. A many-to-one preference of an agent satisfies the *substitutable completability* condition if there is a *substitutable completion*, i.e. a certain 'related' substitutable preference in the many-to-many setting for that agent. The preference of an agent has the *unilateral substitutability* property when the preference exhibits complementarities, if any, of only a certain kind; put differently, there may be certain 'permissible' violations of the substitutability condition.

Allowing for a broader class of complementarities gives the bilateral substitutability condition, and further expanding the 'allowed' set of violations of the substitutability condition yields *weak substitutability* condition (abbreviated as *W.Sub.*). The weak substitutability condition is the strongest known necessary condition[5] for stability. The following description and the two Venn diagrams summarize the relationships between these conditions that are known from the existing literature.

$$\begin{aligned} \textit{substitutability}\ [\text{Sub.}] &\Rightarrow \textit{unilateral substitutability}\ [\text{U.Sub.}] \\ &\Rightarrow \textit{bilateral substitutability}\ [\text{B.Sub.}] \\ \textit{substitutability} &\Rightarrow \textit{substitutable completability}\ [\text{Sub.Comp.}] \\ \textit{bilateral substitutability} &\not\Rightarrow \textit{substitutable completability} \\ \textit{substitutable completability} &\not\Rightarrow \textit{bilateral substitutability} \end{aligned}$$



By proving that unilateral substitutability implies substitutable completability, we are able to provide the following unified Venn Diagram.[6]



[4] All the relevant conditions are defined in section 2.2.

[5] The necessary condition implies that if there is an agent with a preference not satisfying the weak substitutability condition then there exists a setup of weak substitutable preferences for other agents such that no stable matching exists as proved in Hatfield and Kojima (2008).

[6] There are some other recent contributions on this topic. Specifically, cumulative offer revealed bilateral substitutability condition is proposed by Flanagan (2014), and observable substitutability and observable substitutability across workers conditions are proposed by Hatfield et al. (2015). Moreover, Zhang (2016) further expands this domain with a new condition–weak observable substitutability and proves that unilaterally substitutable choice functions that satisfy a condition are substitutably completable.