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Discrete-time option pricing with stochastic liquidity☆

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ABSTRACT

Classical option pricing theories are usually built on the law of one price, neglecting the impact of market liquidity that may contribute to significant bid-ask spreads. Within the framework of conic finance, we develop a stochastic liquidity model, extending the discrete-time constant liquidity model of Madan (2010). With this extension, we can replicate the term and skew structures of bid-ask spreads typically observed in option markets. We show how to implement such a stochastic liquidity model within our framework using multidimensional binomial trees and we calibrate it to call and put options on the S&P 500.

1. Introduction

Classical option pricing theories are usually based on the paradigm of complete and frictionless markets. However, even in financial markets that are considered to be highly competitive, we do observe drops in liquidity, which in times of financial turmoils may be significant and spark concerns among market participants. Liquidity has many different facets. In this paper, we measure liquidity as the spread between bid and ask prices. Illiquid assets are characterized by a high spread. When illiquidity draws a wedge between bid and ask prices, we can no longer rely on the law of one price.

The first attempts to explain bid-ask spreads were made by introducing transaction costs such as commission charges or inventory costs.[1] However, these models often fail to explain the magnitude of the spreads observed in the markets. Especially after the financial crisis of 2008, bid-ask spreads of many assets were persistently high and at a level that cannot be explained by transaction costs alone.[2] A different approach was taken by Madan and Cherny (2010) which is based on theory of conic finance, originating from the work by Cherny and Madan (2009). The basic premise is that the market takes the role of a central counterparty that buys and sells assets from and to investors. The investor buys at the ask price and sells at the bid price. The difference of these prices gives rise to the bid-ask spread observed in financial markets. The central counterparty is viewed as passive in that it does not maximize some utility function, but rather carries out all trades that are acceptable to it.[3]

Madan and Cherny (2010) propose to model market illiquidity by a single market stress level parameter, according to which the market assigns bid and ask prices to assets based on the concept of acceptability indices. This static liquidity model was further extended and taken to the data in Corcuera et al. (2012) and Albrecher et al. (2013). These papers suggests at least two stylized facts for implied liquidity. First, market liquidity implied by real-world data exhibits both a skew and a term structure. This observation is in stark contrast to the assumption of a single liquidity parameter over all maturities and strikes. Second, they show that when we calibrate a single market liquidity parameter for the S&P

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[1] See, e.g., Davis et al. (1993); Shreve and Soner (1994); Soner et al. (1995); Cvitanić and Karatzas (1996); Barles and Soner (1998).

[2] See, e.g., Pedersen (2009) for the average bid-ask spreads of large-cap U.S. stocks from June 2006 to June 2009.

[3] Acceptability itself is measured by acceptability indices, which are rooted in the theory of coherent risk measures as developed in Artzner et al. (1999).

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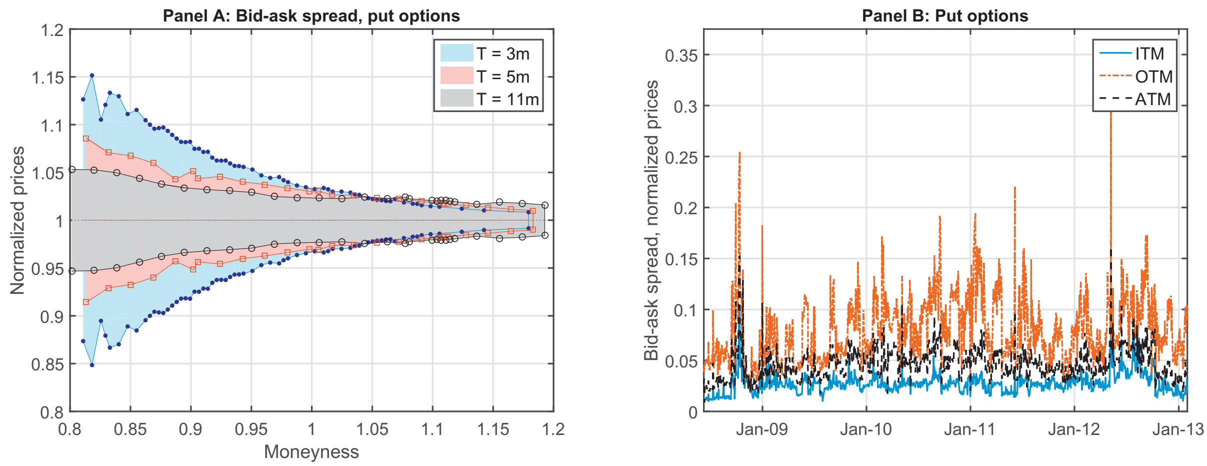

Fig. 1. In Panel A, we plot the bid-ask spread for selected maturity slices of the European puts on the S&P 500 on July 20, 2012. We normalize the bid-ask spreads by the mid-prices. Moneyness is expressed as the ratio of strike over forward price. In Panel B, we plot the time-series of bid-ask spreads of in-the-money (ITM, 120% moneyness), out-of-the-money (OTM, 80% moneyness), and at-the-money (ATM, 100% moneyness) puts on the S&P500 with maturity of 5 months on July 20, 2012.

500 option market, we obtain a time-series of the implied liquidity parameter with a mean-reverting stochastic behavior over time.

These stylized facts are illustrated in Fig. 1. In Panel A, we plot the bid-ask spreads in terms of normalized prices of European puts written on the S&P 500 index. Clearly, bid-ask spreads differ across moneyness and time-to-maturity. In Panel B of Fig. 1, we plot the historical bid-ask spreads for European puts with a maturity of five months. Clearly, these historical spreads change over time and exhibit some mean-reverting behavior. Hence, the empirical evidence presented in Corcuera et al. (2012) and Albrecher et al. (2013), together with the snapshot of historical bid-ask spreads in Fig. 1, provides us with valuable guidance in designing a stochastic liquidity model that may account for the skew and term structure effects of implied liquidity.

We contribute to the steadily growing literature on liquidity modeling for option pricing in two ways. First, by making the setup of the discrete-time constant liquidity model of Madan (2010) more rigorous, we can simplify his results and extend the constant liquidity model to a stochastic liquidity framework. Our Theorem 1 allows us to represent bid and ask prices under stochastic liquidity given by backward recursions as time-consistent and dynamically translation invariant nonlinear expectations. This result opens the door to introduce stochastic liquidity in the conic finance framework. As an illustration, we apply a specific stochastic liquidity model using multidimensional binomial trees to the S&P 500 index option market. We show that this extension improves the fit of the term and skew structures in bid-ask spreads observed in markets. To the best of our knowledge, this is the first model to treat liquidity as a separate process that can be applied to the pricing of bid-ask spreads of derivatives. Compared to other approaches, our model is also suitable for deriving the bid and ask prices of path-dependent options such as Asian and Barrier options.

There have been various other endeavors on how to introduce dynamic bid-ask spreads in option pricing based on the model of Madan and Cherny (2010). An obvious way to do so is to model the bid and ask price as two separate stochastic processes, as suggested in Madan and Schoutens (2014). However, it can be considered a drawback that for payoffs which are not comonotone with a long or short stock position, this approach only gives lower and upper bounds for bid and ask prices. Another avenue is to follow the literature of dynamic risk measures. Mirroring the steps of the static one-step model, Bielecki et al. (2013) define dynamic acceptability indices with the help of dynamic coherent risk measures as discussed in, e.g., Riedel (2004) and Artzner et al. (2007). The disadvantage of using dynamic coherent risk measures is that they are not as tractable and intuitive compared to the static setup. It is furthermore not clear how a stochastic liquidity component could be incorporated. Biagini and Bion-Nadal (2014) tackle the issue in a similar way and arrive at a continuous-time version, while Bielecki et al. (2015) make use of Backward Stochastic Difference Equations (BSΔEs) and Rosazza Gianin and Sgarra (2013) derive dynamic risk measures from g-expectations.

Other than the approaches described above which are all based on or inspired by Conic Finance, there is a large body of literature that explores liquidity and bid-ask spreads in option markets.[4] One way to derive bid and ask prices of derivatives is by considering the replication costs induced by an illiquid underlying. A popular model in this direction was conceived by Çetin et al. (2004) who propose to model illiquidity by assuming that prices of underlyings are provided by a stochastic supply curve, that is not impacted by the actions of buyers and sellers. The resulting bid and ask prices are then dependent on the trade size which differs from our assumption of a trade-invariant bid-ask spread. They find that, in discrete time, hedging derivatives by trading the illiquid underlying incurs liquidity costs. However, results in Çetin et al. (2006) indicate that this approach can only partially explain bid-ask spreads of derivatives observed in the market. In a separate study, Chou et al. (2011) also conclude that it is not sufficient to only consider the underlying's liquidity, but also an option's own liquidity must be taken into account. In contrast, our model does not specifically differentiate between underlying and option liquidity and indeed does not use replicating strategies to derive option prices. Hence, we assume that all information regarding liquidity is contained in the bid and ask prices of the option market.[5]

The rest of the paper is structured as follows. In Section 2, we review the one-period framework of Madan and Cherny (2010). Section 3 introduces the multi-period model with stochastic liquidity. In Section 4, we bring our model to the data and show that the stochastic liquidity model helps to explain the skew and term structure typically observed in options' bid ask spreads. Finally, Section 5 concludes. All proofs are delegated to the appendix.

[4] See, e.g., George and Longstaff (1993); Engle and Neri (2010); Chou et al. (2011); Chan and Chung (2012), Bongaerts et al. (2011); Christoffersen et al. (2015), and Feng et al. (2014), to name a few.

[5] In our approach, we cannot differentiate between how much of the illiquidity reflected in the options' bid-ask spreads are due to the illiquidity of the underlying market and how much is due to the option market itself.