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Market equilibrium under piecewise Leontief concave utilities ☆

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A R T I C L E I N F O

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A B S T R A C T

Leontief is one of the most widely used and extensively studied function in economic modeling, for both production and preferences. However it lacks the desirable property of diminishing returns. In this paper, we consider piecewise Leontief concave (p-Leontief) utility function which consists of a set of Leontief-type segments with decreasing returns and an upper limit on the utility on each segment, and study complexity of computing equilibria in both Fisher and exchange market models. Leontief is a special case of p-Leontief with exactly one segment with no upper limit.

While it is well known that equilibrium computation in Fisher markets with Leontief utilities is polynomial time, in case of p-Leontief, even with two segments, we show that the problem is PPAD-hard. On the other hand for the case when coefficients across segments are scaled versions of each other, we give a polynomial time algorithm. As a corollary, we obtain a non-trivial new class of exchange markets with Leontief utilities where an equilibrium can be computed in a polynomial time, whereas the general problem is known to be PPAD-hard.

For p-Leontief exchange markets with *pairing economy* we show that equilibria are rational and we obtain a simplex-like algorithm to compute one using the classic Lemke–Howson scheme, thereby extending the results of Ye, TCS 2007, and Codenotti, Saberi, Varadarajan and Ye, SODA 2006 for Leontief to p-Leontief utilities.

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1. Introduction

Market equilibrium is a fundamental solution concept in mathematical economics and has been studied extensively since the work of Walras [1]. The notion of equilibrium is inherently algorithmic, with many applications in policy analysis and recently in e-commerce [2–4]. An Arrow–Debreu (exchange) market consists of a set of agents and a set of divisible goods, where each agent has an initial endowment of goods and a utility (preference) function over bundle of goods. At equilibrium, each agent buys a utility maximizing bundle from the money obtained by selling her initial endowment and the market clears.

It is customary in economics to assume utility functions to be concave and satisfying the law of diminishing returns. Leontief utility function [5] is a well-studied concave function, where goods are complementary and they are needed in a fixed proportion for deriving a positive utility, e.g., bread and butter. It is a homogeneous function of degree one, where

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the utility is multiplied by α when the amount of each good is multiplied by α, for any $\alpha > 0$; hence it does not model diminishing returns. Consider the following example.

Example. Suppose Alice wants to consume sandwiches and for making a sandwich, she needs two slices of bread and one slice of cheese. It seems her utility function for bread and cheese can be modeled as a Leontief function, but it is not appropriate because her utility for the second sandwich is less than the first one due to satiation, and so on.

In this paper, we define piecewise Leontief concave (p-Leontief) utility function, which not only generalizes Leontief but also captures diminishing returns to scale and seems to be more relevant in economics. Further, we derive algorithmic and hardness results for both Fisher[1] and exchange market models under these functions. A p-Leontief utility function consists of a sequence of Leontief functions, each with a specified utility limit, where the utility is derived at a decreasing rate as we go from left to right. We will call each Leontief function a segment. Due to diminishing returns property, such a p-Leontief function is concave (see Section 2.4 for the precise definition). Observe that a Leontief function is simply a p-Leontief function with exactly one segment and no utility limit.

Recall the above example. Alice's utility for bread and cheese can be modeled as a p-Leontief function as follows: On the first segment, a unit of utility can be derived by consuming 2 slices of bread and 1 slice of cheese, and the upper limit is 1, i.e., at most 1 unit of utility can be derived on this segment. On the second segment, a unit of utility can be derived by consuming 4 slices of bread and 2 slices of cheese, and the upper limit is 2, and so on.

Since Leontief function is a special case of p-Leontief function, all hardness results for Leontief utilities [6] simply carry over to p-Leontief utilities and we get the following: Computing an equilibrium in an exchange market with p-Leontief utilities is PPAD-hard, and all equilibria can be irrational even if all input parameters are rational numbers.

There is a qualitative difference between the complexity of computing an equilibrium in Fisher and Arrow–Debreu markets under Leontief utilities; while there is a polynomial time algorithm in the former case through Eisenberg's convex program [7], the problem is PPAD-hard in the latter case [6].[2] In contrast, we show that Fisher is no easier than Arrow–Debreu under p-Leontief utilities and obtain the following theorem.

Theorem 1 *(Hardness of Fisher p-Leontief). Computing an equilibrium in Fisher markets with p-Leontief utilities, even with two segments, is PPAD-hard.*

For the above theorem, we essentially give a reduction from exchange p-Leontief with k segments to Fisher p-Leontief with $k+1$ segments. Since the former problem is PPAD-hard even when $k=1$, the result follows. Further we consider an important special case of p-Leontief utilities where coefficients across segments are scaled versions of each other, and we show that in this case Fisher market equilibria can be computed in a polynomial time. This special case may arise in various practical situations, like in the above example of Alice's utility function for sandwiches made of bread and cheese where the proportion of bread and cheese to make a sandwich remains 2:1 (on each segment), however her utility per unit of sandwich decreases.

As a corollary of above results, we obtain a non-trivial new subclass of tractable exchange Leontief markets, *i.e.,* where the sum of endowment matrix (W) and Leontief utility coefficient matrix (U) is a constant times all one matrix (see Section 4.1 for details). We note that Fisher is a special case of exchange market model for which W is of rank one. However an exchange market in our subclass may have an arbitrary W. To the best of our knowledge, apart from the Fisher markets, we are not aware of any other non-trivial tractable classes of exchange Leontief markets.

Pairing economy is a special case of exchange markets, where each agent brings a unique good to the market, *i.e.,* there is one-to-one pairing between agents and goods (see Section 2.1 for precise definition). For the case of pairing economy with Leontief utilities, [6] showed that equilibria are rational and they are in one-to-one correspondence with the symmetric Nash equilibria in a symmetric bimatrix game. We extend the results of [6,9] for Leontief to p-Leontief and obtain the following (informal) theorem.

Theorem 2 *(Pairing economy: rationality and algorithm). In a pairing economy with p-Leontief utilities, equilibrium prices are rational if all input parameters are rational. Further computing an equilibrium is PPAD-complete and there is a finite time algorithm to find one using the classic Lemke–Howson scheme.*

For this, we first characterize equilibrium conditions for exchange market with p-Leontief utilities using the *right* set of variables: a variable to capture price for each good and a variable to capture utility on each segment. In the case of a pairing economy, these conditions can be divided into two parts. The first part captures the utility on each segment, at equilibrium, as a linear complementarity problem (LCP) formulation, where we use the power of complementarity to ensure that segments are allocated in the correct order. The second part is a linear system of equations of type $A\boldsymbol{p} = \boldsymbol{p}$ in prices, given the utilities on each segment.

[1] A special case of exchange market model, defined in Section 2.2.
[2] Containment in PPAD for the Arrow–Debreu Leontief markets is known only for approximate equilibria, while for exact the problem is FIXP-complete [8].

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