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On sunspot fluctuations in variable capacity utilization models[☆]

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ABSTRACT

We investigate the extent to which standard one sector RBC models with positive externalities and variable capacity utilization can account for the large hump-shaped response of output when the model is submitted to a pure sunspot shock. We refine the Benhabib and Wen (2004) model considering a general type of additive separable preferences and a general production function. We provide a detailed theoretical analysis of local stabilities and local bifurcations as a function of various structural parameters. We show that, when labor is infinitely elastic, local indeterminacy occurs through Flip and Hopf bifurcations for a large set of values for the elasticity of intertemporal substitution in consumption, the degree of increasing returns to scale and the elasticity of capital–labor substitution. Finally, we provide a detailed quantitative assessment of the model and conclude with mixed results. We show that although the model is able theoretically to generate a hump-shaped dynamics of output following an i.i.d. sunspot shock under realistic parameter values, the hump is too persistent for the model to be considered fully satisfactory from an empirical point of view.

1. Introduction

In this paper, we emphasize the link between demand shocks and expectation-driven fluctuations based on the existence of sunspot equilibria. More precisely, we investigate the extent to which standard one-sector sunspot models with positive externalities and variable capacity utilization can account for "boom-bust cycles" characterized by procyclical covariations of most macroeconomic variables and a hump-shaped output response when the model is submitted to a pure sunspot shock.

The traditional view put forward in the DSGE literature is that fluctuations are triggered by shocks on economic fundamentals. However, since Cass and Shell (1983), a field of economic research has been developed to analyze the role of agents' expectations in the understanding of macroeconomic fluctuations. In particular, researchers have highlighted the fact that agents can collectively change their expectations due to exogenous reasons, not necessarily related to economic fundamentals. In turn, these changes in expectations generate fluctuations which validate ex-post the initial expectations and are thus consistent with rational expectations, i.e. sunspot fluctuations are based on self-fulfilling prophecies.

The first sunspot model using the framework of the RBC/DSGE literature (Benhabib and Farmer, 1994) was shown to perform as well as, or even better than, the canonical RBC model (Farmer and Guo, 1994). However, a major hurdle this literature faced was that the existence of sunspot equilibria required very large levels of increasing returns to scale, inconsistently with the data. This weakness was considered one of the main challenge for the macroeconomic sunspot literature until Wen (1998) proposed a simple extension consisting in introducing a variable capital utilization rate in the Benhabib–Farmer setup, in the spirit of Greenwood et al. (1988).[1] It was shown that this simple extension to the canonical one-sector model was sufficient to allow for the existence of sunspot fluctuations under low and empirically plausible levels of

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1 An alternative explanation is to introduce a two-sector setup with increasing returns affecting mostly the investment good sector. See Dufourt et al. (2015).

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increasing returns. Moreover, Benhabib and Wen (2004) showed that this model could also explain many dimensions of observed business cycles when the model is submitted to correlated fundamental and sunspot shocks. In particular, the model is able to account for Pigou cycles: periods of booms and busts triggered by exogenous changes in agents' expectations and affecting most macroeconomic variables. The Benhabib–Wen (henceafter BW) model then put an end to years of discussions about the credibility of sunspot models and their ability to explain salient features of observed business cycles.

Yet, a careful examination of the results presented by BW reveals that there remains one dimension for which the model is not entirely satisfactory. While a positive sunspot shock does generate procyclical movements in consumption, hours worked, investment and output – consistently with the data – these impulse responses are not *hump-shaped*. This is problematic since, starting with the seminal analysis of Blanchard and Quah (1989), there exists a bulk of empirical literature showing that the typical impulse response of output to a properly defined (through various assumptions) "demand shock" is hump-shaped. Clearly, for an explanation of actual business cycles based on sunspot/self-fulfilling prophecies to be fully convincing, these models should be able to replicate all the main stylized facts associated with a canonical demand shock identified in the empirical literature.

The aim of this paper is thus twofold. First, we observe that in the initial BW model, very tight restrictions on the specification of preferences and on the production side of the economy are considered. These restrictions imply in turn very specific values for some crucial economic parameters that are known to affect not only the local stability properties of the models, but also their business cycle properties: the elasticity of intertemporal substitution (EIS) in consumption, the degree of increasing returns to scale (IRS), the wage-elasticity of labor supply, and the capital–labor elasticity of substitution in production. From a theoretical point of view, it is thus important to assess whether the result that indeterminacy can occur under low degrees of increasing returns to scale in the BW setup is robust when we consider the whole range of empirically credible values for these parameters. As a result, we provide in the first part of the paper a complete analysis of the local stability properties of the model as a function of these various economic parameters.

Second, based on the whole picture of the ranges of values for which the model is locally indeterminate, we assess whether the inability of the BW model to replicate a hump-shaped output dynamics in response to a pure sunspot shock is *robust* – i.e., structural to the model – or if it is due to the fact that this model was evaluated under too strong restrictions regarding the specifications of individual preferences and the production function.

Our main findings can be summarized as follows. First, we prove that, under the class of general additively separable preferences and a general production function, local indeterminacy occurs through Flip and Hopf bifurcations for a large set of values for the degree of IRS, the EIS in consumption and the capital–labor elasticity of substitution, provided that the labor supply elasticity is large. In particular, the degree of IRS can be made arbitrarily small when the other parameters are in an appropriate range. Likewise, indeterminacy can occur for a range of values for the capital–labor elasticity of substitution that extends well beyond one – including, when the degree of IRS is not too large, the case a perfect factor complementarity. Second, we perform a quantitative analysis of the model directed toward the ability to replicate a hump-shaped dynamics of output in response to a pure sunspot shock. We show that, from a theoretical point of view, a standard one-sector model with variable capacity utilization in the spirit of BW *is able* to reproduce such a hump-shaped dynamics, while maintaining the procyclicality of all the main macroeconomic variables along the business cycle (boom-bust cycles). The key ingredients for obtaining this result are to consider a value for EIS in consumption in the upper range of available empirical estimates, a quite substantial increase in the degree of factor substitutability compared to the Cobb–Douglas production function, and a slightly larger degree of IRS than considered in the BW model. On the other hand, we also show that the obtained hump-shaped dynamics is too persistent to be considered entirely consistent with observed data, leading us to conclude that the puzzle is improved but not entirely solved.

This remaining of this paper is organized as follows. We present a generalized version of the one-sector model with variable capital utilization rate in Section 2, as well as the corresponding intertemporal equilibrium and steady state. We derive the local stability properties and local bifurcations in Section 3. In Section 4 we discuss the ability of our model to account for the stylized facts associated with a canonical demand shock when the source of the business cycle is a pure sunspot shock. We also check the robustness of our results considering extended formulations with habit formation in consumption or dynamic learning by doing in production. We conclude in Section 5.

2. The model

We consider a closed economy framework in the spirit of Wen (1998) and Benhabib and Wen (2004). The economy is composed of a large number of identical infinitely-lived agents and a large number of identical producers. Agents consume, supply labor and accumulate capital subject to a variable capacity utilization rate that also influences the depreciation rate of capital. Firms produce the unique final good which can be used either for consumption or investment. All markets are perfectly competitive, but there are externalities in production.

2.1. The production structure

The production sector is composed of a large number of identical firms which operate under perfect competition. Output Y_t is produced by combining labor L_t and capital services u_tK_t, where u_t is the capital utilization rate. The technology of each firm exhibits constant returns to scale with respect to its own inputs and we consider that knowledge diffusion occurs, in the sense that each of the many firms benefits from positive externalities due to the contribution of the average level of labor $\bar{L}$ and capital services $\bar{u}\bar{K}$. These external effects are exogenous and not traded in markets. The production function is

$$Y_t = Af(u_tK_t, L_t)e(\bar{u}_t\bar{K}_t, \bar{L}_t) \tag{2.1}$$

where $A > 0$ is a scaling technology parameter and $e(\bar{u}_t\bar{K}_t, \bar{L}_t)$ is the externality variable. Our first departure from BW is that we do not restrict the production function to be Cobb–Douglas. Rather, our production function is general and satisfies:

Assumption 1. $f(uK, L)$ is $\mathbf{C}^2$ over $\mathbb{R}^2_{++}$, increasing in (uK, L), concave over $\mathbb{R}^2_{++}$ and homogeneous of degree one. $e(\bar{u}\bar{K}, \bar{L})$ is $\mathbf{C}^1$ over $\mathbb{R}_{++}$ and increasing in $(\bar{u}\bar{K}, \bar{L})$.

Firms rent effective capital units at the real rental rate r_t and hire labor at the unit real wage w_t. The profit maximization program of the firm,

$$\max_{\{Y_t, L_t, u_tK_t\}} Y_t - w_tL_t - r_tu_tK_t,$$

leads to the standard demand function for effective capital u_tK_t and labor L_t:

$$r_t = Af_1(u_tK_t, L_t)e(\bar{u}_t\bar{K}_t, \bar{L}_t) \tag{2.2}$$

$$w_t = Af_2(u_tK_t, L_t)e(\bar{u}_t\bar{K}_t, \bar{L}_t). \tag{2.3}$$