Journal of Mathematical Behavior xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Students' epistemological frames and their interpretation of lectures in advanced mathematics

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ARTICLE INFO

Keywords:
Epistemological frames
Real analysis
Student understanding of lecture

ABSTRACT

In this paper, we present a comparative case study of two students with different epistemological frames watching the same real analysis lectures. We show that students with different epistemological frames can interpret the same lecture in different ways. These results illustrate how a student's interpretation of a lecture is not inherently tied to the lecture, but rather depend on the student and her perspective on mathematics. Thus, improving student learning may depend on more than improving the quality of the lectures, but also changing student's beliefs and orientations about mathematics and mathematics learning.

1. Introduction

In recent years, several researchers have explored the relationship between students' epistemological beliefs and their learning of advanced mathematics. In particular, some scholars have claimed that some students struggle to learn mathematics because they lack the epistemological beliefs to support this learning (e.g., Alcock & Simpson, 2004; Bressoud, 2016; Dawkins & Weber, 2017; Lew, Fukawa-Connelly, Mejía-Ramos, & Weber, 2016; Solomon, 2006). The primary aim of this paper is to extend this research. In particular, we adapt the notion of epistemological frames (e-frames), a construct from physics education (e.g., Redish, 2004), and illustrate how students who hold different e-frames can interpret the same advanced mathematical lecture in different ways. In particular, we first give an account of two students' e-frames in an advanced mathematical setting; we then use these e-frames to give a fine-grained account for these students' different interpretations of the same utterances by a lecturer.

2. Theoretical perspective and related literature

2.1. Epistemological frames

Goffman (1997) introduced the notion of *frame* to describe how individuals develop expectations to help them make sense of the complex social spaces that they inhabit. For instance, most adults in the Western world have a "restaurant frame" consisting of expectations that are activated when they enter a restaurant. When frequenting a restaurant, an individual likely would expect that the restaurant employees will prepare food for the individual, the individual will be obligated to pay for this food, and so on (Schank, 1990). Such restaurant frames are usually helpful; these frames allow individuals to act sensibly in restaurants that they have never visited before. However, frames can occasionally be counterproductive if two individuals frame the same situation in different ways. For instance, a European diner may offend a waiter in the United States if she was not aware of the United States custom to leave at

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https://doi.org/10.1016/j.jmathb.2017.12.001
Received 3 August 2017; Received in revised form 31 October 2017; Accepted 7 December 2017
0732-3123/

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Physics educators have introduced the notion of an individual's *epistemological frame*, or e-frame, as consisting of their epistemological expectations about a pedagogical situation. These consist of an individual's responses to questions such as "what do I expect to learn?" and the related questions of "what counts as knowledge or an intellectual contribution in this environment?" and "by what standards will intellectual contributions be judged?" (Redish, 2004). If a teacher and her students approach the same pedagogical activity with different e-frames, the students likely will not learn what their teacher intends. For instance, Redish (2004) described a physics tutorial in which students were asked to form a hypothesis. The teacher's aim of this activity was for students make qualitative predictions using their conceptual understanding of physics principles. Redish found that a student who viewed intellectual contributions in physics as consisting of numeric answers derived from textbook formulas responded to such tasks by engaging in computations, thereby avoiding the conceptual considerations the activity was designed to elicit.

We are not aware of any mathematics education research that has specifically used the notions of e-frames to account for students' behaviors. However scholars have explored the relationship between students' epistemological beliefs and their concomitant mathematical cognition. In many of these cases, the claims of these scholars can be expressed using the construct of e-frames. For instance, Thompson (2013) presented a situation in which a teacher provided many conceptual explanations to her high school algebra class but these explanations were ignored by some students in the class who had a procedural orientation. We might interpret Thompson's claim with e-frames as follows. In the students' e-frames, an intellectual contribution in an algebra class consisted of using a procedure to solve a problem symbolically. The teacher viewed part of the intellectual contribution of her presentation as explaining the meaning of the procedure that she was implemented. Since students did not recognize this as a legitimate intellectual contribution in a mathematics class, they simply ignored the conceptual explanations.

We can use similar reasoning to characterize other mathematical constructs. For instance, the didactical contract (Brousseau, Sarrazy, & Novatna, 2014) includes expectations about what mathematical contributions the teacher is required to make, establishing sociomathematical norms (Yackel & Cobb, 1996) involves the negotiation of what an acceptable mathematical contribution is, and institutional meanings of proof (Recio & Godino, 2001) are expectations about what constraints a justification must satisfy in different contexts. In summary, while we are introducing the notion of e-frames to mathematics community, this work builds upon a rich tradition of scholarship examining the links between students' epistemology and cognition. Our contribution is offering a more fine-grained account of how specific e-frames influence students' interpretations of specific mathematical utterances in advanced mathematics.

2.2. Logical versus psychological understandings in advanced mathematics

In this paper, we distinguish between two ways of knowing a mathematical concept. An individual knows a concept *psychologically* if she believes the statement is true and feels that they understand why the statement is true. An individual knows a concept *logically* if she can provide a deductive justification demonstrating the statement is true from previous statements (usually definitions and axioms) that are assumed to be true.

We make three observations about this distinction. First, in many mathematical settings, psychological and logical knowing are inextricably intertwined. Mathematicians often believe a statement is true exactly when they see how it can be logically deduced from other things that are known or assumed to be true (e.g., Harel & Sowder, 2007). Second, psychological knowing and logical knowing are nonetheless distinct. Some mathematicians hold rational certainty in the veracity of unproven conjectures (e.g., Goldbach's conjecture) and others retain some doubt in claims that have been proven (on the grounds that they cannot be certain that their proofs are correct) (c.f., Weber, Inglis, & Mejia-Ramos, 2014). This reflects the view that the acceptability of a proof is dependent upon a reference theory specifying what facts are allowed to be assumed (Mariotti, 2006). Third, in some cases, the purpose of proof is not to enhance one's psychological knowing, but to provide logical justification (c.f., Dawkins & Weber, 2017). Mamona-Downs and Downs (2010) expressed this clearly when she wrote that "the point [of proof] is not so much about conviction, but how we can clarify the bases of the reasoning employed" (p. 2338). Arzarello (2007) expressed a similar sentiment, arguing that the purpose of proof is to give meaning to a statement by placing the statement into a network of mathematical knowledge in the form of logical consequence.

2.3. Systematization

In this paper, we focus on a particular type of activity that DeVilliers (1990) has coined *systematization*. In this activity, mathematicians transform an existing theory—i.e., a constellation of concepts and related statements that are accepted as true—into a unified whole. Mathematicians do so by creating a system of axioms and definitions and then demonstrating that commonly accepted statements within the existing theory are deductive consequences from this system of axioms and definitions. As DeVilliers (1990) noted, with systematization, "the main objective is clearly not *to check whether statements are really true*" (p. 21, emphasis was the author's). In our interpretation, the purpose of systematization is not to enhance one's *psychological knowledge*; the statements being justified are already accepted as true. Rather, the purpose is to create a system of axioms and definitions that lets us provide logical justifications for things that are accepted as true. Dawkins and Weber (2017) claimed that mathematicians desire justification in an *a priori* manner so that the non-trivial claims that mathematicians accept as true can be seen as necessary logical consequences of how various terms are defined. Hence, systematizing a theory is useful as it determines the specified starting points from which the rest of the theory can be deduced. The authors further remarked that students might not perceive the value of *a priori* knowledge and therefore might not see the value of providing explicit unambiguous definitions.