Computers and Operations Research 80 (2017) 147–158

Contents lists available at ScienceDirect

Computers and Operations Research

journal homepage: www.elsevier.com/locate/cor

Maximizing the number of conflict-free aircraft using mixed-integer nonlinear programming

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ARTICLE INFO

Article history:
Received 20 September 2015
Revised 17 October 2016
Accepted 2 December 2016
Available online 5 December 2016

Keywords:
Mixed-integer nonlinear programming
Conflict detection and resolution
Air traffic control
Deterministic global optimization
Modeling

ABSTRACT

We address the conflict detection and resolution problem in air traffic control, where an aircraft conflict is a loss of separation between aircraft trajectories. Conflict avoidance is crucial to ensure flight safety and remains a challenging traffic control problem. We focus on speed control to separate aircraft and consider two approaches: (i) maximize the number of conflicts resolved and (ii) identify the largest set of conflict-free aircraft. Both problems are modeled using mixed-integer nonlinear programming and a tailored greedy algorithm is proposed for the latter. Computational efficiency is improved through a pre-processing algorithm which attempts to reduce the size of the conflict resolution models by detecting the existence of pairwise potential conflicts. Numerical results are provided after implementing the proposed models and algorithms on benchmark conflict resolution instances. The results highlight the benefits of using the proposed pre-processing step as well as the versatility and the efficiency of the proposed models.

1. Introduction

One of the missions of air traffic control services is to ensure the safety of aircraft throughout their flight. This task is achieved by continuously monitoring aircraft trajectories, anticipating potential loss of separation, known as *conflict*, and issuing appropriate conflict resolution maneuvers. Due to the forecasted increase in flight travel volume in the next decades (see, e.g., [12]), the automation of conflict detection and resolution procedures has received a growing attention over the past few years. Indeed, the traffic growth is likely to impact air traffic controllers workload, with consequent safety issues which could lead to considerable delays.

Aircraft conflict avoidance can be achieved through a variety of maneuvers to separate aircraft trajectories. Flight heading angle deviation is the most widely used separation maneuver. Alternatively, flight level changes can be performed, even if they are usually not preferred due to passenger comfort and fuel consumption considerations. Another possibility is to adjust flights' speed subject to aerodynamical and passenger comfort constraints. In contrast to other separation maneuvers, speed control often requires some level of automation to be operated due to the (small) magnitude of the possible speed adjustments and is therefore seldom used by air traffic controllers ([15]). See [16] for a review up to the year 2000 of mathematical models for aircraft conflict avoidance.

The approach proposed in the present work aims at performing conflict avoidance by aircraft speed control and is based on mixed-integer optimization, which is attracting a growing attention in Air Traffic Management (ATM), in particular for air conflict detection and resolution problems. First approaches of this kind date back to 2002 ([18,22]), where mixed-integer linear programs were proposed, based on aircraft separation by heading or speed changes, derived from geometrical considerations on aircraft trajectories. More recently, the works of [2–4] extend the models of [18] through mixed-integer nonlinear programs based on various aircraft separation techniques. [17] proposes a mixed-integer linear programming model for air conflict resolution which combines speed and heading maneuvers and relies on a space-discretization of aircraft trajectories. Mixed-integer nonlinear programming for aircraft speed change maneuvers executed in time windows is proposed in [10]. Recent works that focus exclusively on speed regulation maneuvers also include [19–21], where a space-discretization approach is used to represent aircraft trajectories and separation. An overview of MINLP modeling for aircraft conflict avoidance is presented in [9].

In most of the above works, it is implicitly assumed that the range of the selected separation maneuvers is broad enough to re-

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[1] Financial support by French National Research Agency (ANR) through grant ANR 12-JS02-009-01 "ATOMIC" is gratefully acknowledged.

http://dx.doi.org/10.1016/j.cor.2016.12.002
0305-0548/

solve all conflicts. In this context, the aim is generally detecting and solving all conflicts that may occur in the monitored airspace during a time horizon, and the focus is to optimize the amount of resources—typically minimizing the deviation to aircraft nominal trajectories—used for deconfliction. However, if one restricts himself to a subset of possible separation maneuvers, it may happen that not all conflicts can be resolved. For instance, if the available separation maneuvers are limited (e.g. finite number of alternative trajectories) or if their range is tightly constrained (as in the case of speed deviations), and the number or the severity of the potential conflicts is significant (e.g. high aircraft density), then the selected separation maneuvers may not suffice to ensure the feasibility of the corresponding optimization problem where one aims at separating all aircraft. Hence, it can be of interest to optimize the number of conflicts that can be solved when a separation maneuver is applied, in order to easily distinguish between conflicts that can be solved and those that require the application of another separation strategy. This lead in [8,19–21] to propose objective functions and models seeking to maximize the number of conflicts resolved or to minimize the total conflict duration.

The interest for this kind of approaches specially applies to deconfliction based on speed control. Speed control-based conflict resolution models emerged at the beginning of the XXI century with the advent of advanced flight management systems that provided more accurate trajectory predictions [25]. Building on this technological development, speed control methods offered a new way to improve traffic conditions by adjusting aircraft future longitudinal positions. Soon after, the concept of *subliminal speed control* was introduced on the premise that minor speed adjustments around aircraft nominal speeds were almost imperceptible to air traffic controllers. This paradigm has been validated in field experiments [11] in the context of the ERASMUS ([7]) project. Due to its limited impact on air traffic controllers' workload ([5]), conflict avoidance by subliminal speed control is considered a promising approach to introduce some automation in ATM in the near future to improve traffic conditions using a very moderate amount of control resources. It constitutes a competitive candidate to introduce a "filter" on air traffic, prior to the use of more trajectory-intensive conflict avoidance methods (e.g. heading and altitude maneuvers) that would require either controller intervention or a fully automated traffic control paradigm. From a practical point of view, the approach proposed in this paper can be intended as a first step, acting as a pre-processing, to efficiently perform conflict avoidance.

In this paper, building on [8], we first present a model from mixed-integer nonlinear programming (MINLP) to maximize the number of solved conflicts (pairwise separated aircraft) when this maneuver is applied. This model, following [10], does not require any form of discretization, unlike the most of previous works in the area, thus resulting in a compact mixed-integer nonlinear program. The computational performance of such a MINLP is improved by a novel technique to identify potential aircraft conflicts, based on the solution of a simple concave bound-constrained continuous maximization problem. The detection of potential conflicts is carried out in a pre-processing phase where pairwise conflicts are identified before a complete model, *i.e.* with all the relevant pairs of aircraft, is solved. Then, based on the observation that for a large number of aircraft in the observed air sector it may be difficult to solve many conflicts simultaneously, and that, even not imposing solving all conflicts, the problem can be computational demanding, we propose a novel approach to identify the largest set of conflict-free aircraft. To this aim, we first propose a MINLP model and then, to improve its computational sustainability, a greedy algorithm that iteratively removes conflicting aircraft from the original set containing all the observed aircraft. Numerical experiments show that the proposed approach allows us to find competitive solutions.

The paper is organized as follows. In Section 2 we first discuss about the mathematical representation of aircraft trajectories separation, then we propose a mathematical programming formulation, from MINLP, for aircraft conflict avoidance achieved through speed regulation. Section 3 focuses on conflict detection, and presents an algorithm to identify potential aircraft conflicts when speed control is applied. In Section 4 we propose an approach to identify the largest set of aircraft that are conflict-free when speed regulation is applied. This is done through a MINLP problem, as well as a greedy aircraft-removal algorithm tailored on the problem. In Section 5 we present and discuss the results of numerical experiments carried out to validate the proposed approaches. Section 6 draws some conclusions and presents some perspectives.

2. Problem modeling

In this section, we first discuss the mathematical representation of the crucial condition of separation between aircraft trajectories. Then, we propose a mathematical programming formulation for aircraft conflict avoidance, where the above separation condition clearly represents the main constraint on the decision variables.

2.1. Representation of aircraft separation

Let us consider a set A of n aircraft flying during their cruise flight in a given air sector, all at the same flight level. Let $\mathbf{x}_i(t)$ be the vector representing the position of flight i at time t. The relative position of aircraft i and j at time t can be represented as

$$\mathbf{x}^r_{ij}(t) = \mathbf{x}_i(t) - \mathbf{x}_j(t).$$

Let d be the horizontal separation standard, aircraft i and j are separated if

$$\|\mathbf{x}^r_{ij}(t)\| \geq d, \forall t \tag{1}$$

where $\|\cdot\|$ is the Euclidean norm in the two-dimensional space formed by aircraft trajectories. Assuming that uniform motion laws apply, $\mathbf{x}^r_{ij}(t)$ can also be expressed as the sum of the initial relative position of flights i and j, $\mathbf{x}^{r0}_{ij}$, and their relative speed, $\mathbf{v}^r_{ij}$: $\mathbf{x}^r_{ij}(t) = \mathbf{x}^{r0}_{ij} + \mathbf{v}^r_{ij}t$. Therefore squaring Eq. (1) we obtain the separation condition

$$f_{ij}(t) \equiv \|\mathbf{v}^r_{ij}\|^2 t^2 + 2\mathbf{x}^{r0}_{ij} \cdot \mathbf{v}^r_{ij} t + \|\mathbf{x}^{r0}_{ij}\|^2 - d^2 \geq 0 \tag{2}$$

where $\cdot$ is the inner product in the Euclidean space. From Eq. (2), $f_{ij}(t)$ is a 2nd order convex polynomial in t which is minimal when its derivative vanishes:

$$f'_{ij}(t) = 0 \quad \Leftrightarrow \quad t^m_{ij} = \frac{-\mathbf{x}^{r0}_{ij} \cdot \mathbf{v}^r_{ij}}{\|\mathbf{v}^r_{ij}\|^2} \tag{3}$$

t^m_{ij} is the time at which $f_{ij}(t)$ is minimal and therefore represents the instant at which the aircraft relative position is minimal. Following [10], we observe that, substituting t^m_{ij} in $f_{ij}(t)$, the separation condition (2) can be represented as

$$\|\mathbf{v}^r_{ij}\|^2(\|\mathbf{x}^{r0}_{ij}\|^2 - d^2) - (\mathbf{x}^{r0}_{ij} \cdot \mathbf{v}^r_{ij})^2 \geq 0. \tag{4}$$

that does not depend anymore on t.

Observe that the discriminant of the 2nd order equation $f_{ij}(t) = 0$, Δ_{ij}, is equal to

$$\Delta_{ij} = 4(\mathbf{x}^{r0}_{ij} \cdot \mathbf{v}^r_{ij})^2 - 4\|\mathbf{v}^r_{ij}\|^2(\|\mathbf{x}^{r0}_{ij}\|^2 - d^2), \tag{5}$$

so the following cases can occur:

1. if $\Delta_{ij} \leq 0$, then $f_{ij}(t)$ has no roots and therefore $\|\mathbf{x}^r_{ij}(t)\| \geq d, \forall t$, hence aircraft are separated