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A branching process approach to power markets

Ying Jiao^a, Chunhua Ma^b, Simone Scotti^c, Carlo Sgarra^{d,*}^a Université Claude Bernard-Lyon 1, Institut de Science Financier et d'Assurances, France^b Nankai University, School of Mathematical Sciences, China^c Université Paris Diderot-Paris 7, Laboratoire de Probabilités et Modèles Aléatoires, France^d Politecnico di Milano, Dipartimento di Matematica, Italy

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ABSTRACT

We propose and investigate a market model for power prices, including most basic features exhibited by previous models and taking into account self-exciting properties. The model proposed extends Hawkes-type models by introducing a twofold integral representation property. A Random Field approach was already exploited by Barndorff-Nielsen et al., who adopted the Ambit Field framework for describing the power price dynamics. The novelty contained in our approach consists of combining the basic features of both Branching Processes and Random Fields in order to get a realistic and parsimonious model setting. We shall provide some closed-form evaluation formulae for forward contracts. We discuss the risk premium behavior, by pointing out that in the present framework, a very realistic description arises. We outline a possible methodology for parameters estimation. We illustrate by graphical representation the main achievements of this approach.

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1. Introduction

Energy markets, and in particular, electricity markets, exhibit very peculiar features. The historical series of both futures and spot prices include seasonality, mean-reversion, spikes and small fluctuations.

After the pioneering paper by Schwarz (1997), where an Ornstein-Uhlenbeck dynamics is assumed to describe the spot price behavior, several different approaches have been investigated in order to describe the price evolution. A comprehensive presentation of the literature until 2008 is offered in the book by Benth et al. (2008).

High frequency trading, on the other hand, introduced some new features in commodity prices dynamics: in the paper by Filimonov et al. (2014) evidence is shown of endogeneity and structural regime shift, and in order to quantify this level the branching ratio is adopted as a measure of this endogenous impact and a Hawkes processes dynamics is assumed as a reasonable modeling framework taking into account the self-exciting properties (Bacry et al., 2015).

Among the several different approaches proposed in the literature in order to describe the spiking phenomenon characterizing the power price dynamics we mention the paper by Lucheroni (2007), where two alternative models based on Stochastic FitzHugh-Nagumo system are introduced. By exploiting the stochastic resonating properties of the solutions in a suitable regime, the author is able to explain several basic features of the power market behavior. Moreover, a systematic investigation performed on the Alberta Power Market data from April 7, 2002 to April 6, 2007, exhibits a very good fitting of the model to real data;

* Corresponding author.

E-mail addresses: ying.jiao@univ-lyon1.fr (Y. Jiao), mach@nankai.edu.cn (C. Ma), scotti@math.univ-paris-diderot.fr (S. Scotti), carlo.sgarra@polimi.it (C. Sgarra).

in particular, the interspike distribution described by both models seems to reproduce very well the spike frequency observed.

Barbu et al. (2015), and Cordoni and Di Persio (2017) provided recently some results on an optimal control problem for the stochastic FitHugh-Nagumo model, pointing out the impossibility to apply standard optimal control methods due to the cubic nonlinearity of the system and making the financial applicability of this model quite challenging, especially considering that some of the most extensively traded options on energy markets, that is swing options, belong, roughly speaking, to the class of American options.

In contrast with the papers just cited, the main focus of the present work is on the jump clustering phenomenon, which is another puzzling feature exhibited by spiking patterns. The purpose of the present paper is to propose a new modeling framework including all the above-mentioned features, still keeping a high level of tractability. The model considered allows to obtain the most common derivatives prices in closed or semi-closed form. Here with semi-closed we mean that the Laplace transform of the derivative price admits an explicit expression.

Kiesel and Paraschiv (2016) presented recently a systematic empirical investigation of electricity intraday prices and suggested an approach based on Hawkes processes in order to describe the power price dynamics and the jump clustering feature. Self-exciting features in electricity prices attracted already some attention by several authors: although the above-mentioned paper by Filimonov et al. (2014) did not investigate endogenous effects in power markets, Herrera and Gonzalez (2014) proposed a self-excited model for electricity spot prices, while Christensen et al. (2009), Clements et al. (2013) pointed out that time between spikes has a significant impact on the likelihood of future occurrences, thus providing a strong support to models including self-exciting properties.

The models we are going to introduce can describe the price dynamics in two different forms, that can be proved to be equivalent: the first is a representation based on random fields, the second is based on Continuous State Branching Processes with Immigration (CBI in the following). The idea of adopting a random fields framework for power prices description is not new: O.E. Barndorff-Nielsen, F.E. Benth and A. Veraart introduced the Ambit Fields to this end, showing how this approach can provide a very flexible and still tractable setting for derivatives pricing (Barndorff-Nielsen et al., 2013, 2014). Moreover CBI processes exhibit the Markov property, while Hawkes processes enjoy this property only in particular cases, and this feature makes our approach more appealing.

A model based on CBI has been proposed recently by Y. Jiao, C. Ma and S. Scotti in view of short interest rate modeling, and in that paper it was shown that, with a suitable choice of the Lévy process driving the CBI dynamics, the model can offer a significant extension of the popular CIR model (Jiao et al., 2017).

The model we propose extends in different ways some relevant models already available in the literature. It belongs to the class of arithmetic models (following the classification proposed by F.E. Benth, J. Salathyte-Benth and S. Koekebakker), and the driving processes are Lévy processes with positive jumps, i.e. subordinators, so it extends the model proposed by Benth et al. (2007) by formulating the dynamics via a random field approach, which allows to include some self-exciting features. On the other hand, the random field approach highlights some similarities with the Ambit Field-based models introduced by Barndorff-Nielsen et al. (2014); the main difference between the model proposed in this paper and the Ambit Field-based models consists of the character of the extra dimension appearing in the random field adopted: while in the Ambit Field setting the parameter of this dimension is a time parameter, in the present setting this will be a parameter

of space type. This main difference will be reflected moreover in the integration domain of the integrals defining the dynamics.

The features of our modeling approach just outlined, allow to introduce the so-called self-exciting properties in a simple and natural way and, although the pricing formulas for basic contracts like forward will exhibit very small changes with respect to those obtained for the previous models, the present model will exhibit a substantially different risk premium term structure.

The paper is organized as follows: in Section 2 we introduce the market model we are going to consider, while in Section 3 we discuss the relations between our model and the CBI processes. In Section 4 we derive some closed formulas for Futures and Option prices when the underlying dynamics is assumed to be given by the model introduced. Section 5 includes a theoretical analysis of the jumps behavior and the self-exciting property. In Section 6 we provide some evidence of clustering behavior of power prices based on real data. In the final section we provide some concluding remarks and discuss futures extensions of the present work.

2. The modeling framework

2.1. Model setup

We now introduce our modeling framework for the electricity price, which is based on stochastic differential equations driven by Lévy random field. We consider a Lévy random field which is a combination of a Gaussian random measure W and a compensated Poisson random measure N independent to W . For background for such general stochastic equations with jumps, we refer the readers to Dawson and Li (2012), Li and Ma (2015), Walsh (1980).

Let us briefly present the preliminaries. We fix a probability space $(\Omega, \mathcal{A}, \mathbb{P})$. A white noise W on \mathbb{R}_+^2 is a Gaussian random measure such that, for any Borel set $A \in \mathcal{B}(\mathbb{R}_+^2)$ with finite Lebesgue measure $|A|$, $W(A)$ is a normal random variable of mean zero and variance $|A|$; and that if A_1, \dots, A_n are disjoint Borel sets in $\mathcal{B}(\mathbb{R}_+^2)$, then $W(A_1), \dots, W(A_n)$ are mutually independent. We denote by N the Poisson random measure on \mathbb{R}_+^3 with intensity λ which is a Borel measure on \mathbb{R}_+^3 in form of the product of the Lebesgue measure on $\mathbb{R}_+ \times \mathbb{R}_+$ with a Borel measure μ on \mathbb{R}^+ such that $\int_0^\infty (\zeta \wedge \zeta^2) \mu(d\zeta) < +\infty$. Note that μ is a Lévy measure since $\int_0^\infty (1 \wedge \zeta^2) \mu(d\zeta) < +\infty$. Recall that for each Borel set $A \in \mathcal{B}(\mathbb{R}_+^3)$ with $\lambda(A) < +\infty$, the random variable $N(A)$ has the Poisson distribution with parameter $\lambda(A)$; moreover, if A_1, \dots, A_n are disjoint Borel sets in $\mathcal{B}(\mathbb{R}_+^3)$, then $N(A_1), \dots, N(A_n)$ are mutually independent. We let $\tilde{N} = N - \lambda$ be the compensated Poisson random measure on \mathbb{R}_+^3 associated to N .

We introduce the filtration $\mathbb{F} = (\mathcal{F}_t)_{t \geq 0}$ as the natural filtration generated by the Lévy random field and satisfying the usual conditions, namely, for any Borel subset $A \in \mathcal{B}(\mathbb{R}_+)$ and $B \in \mathcal{B}(\mathbb{R}_+^2)$ of finite Lebesgue measure, the processes $(W([0, t] \times A), t \geq 0)$ and $(\tilde{N}([0, t] \times B), t \geq 0)$ are \mathbb{F} -martingales.

We consider the following stochastic differential equation in the integral form.

Definition 2.1. Let $a, b, \sigma, \gamma \in \mathbb{R}_+$ be constant parameters. Consider the equation

$$Y(t) = Y(0) + \int_0^t a(b - Y(s))ds + \sigma \int_0^t \int_0^{Y(s)} W(ds, du) + \gamma \int_0^t \int_0^{Y(s-)} \int_{\mathbb{R}_+^3} \zeta \tilde{N}(ds, du, d\zeta) \quad (2.1)$$

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