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PRICING CATASTROPHE OPTIONS WITH COUNTERPARTY CREDIT RISK IN A REDUCED FORM MODEL*

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Abstract In this paper, we study the price of catastrophe options with counterparty credit risk in a reduced form model. We assume that the loss process is generated by a doubly stochastic Poisson process, the share price process is modeled through a jump-diffusion process which is correlated to the loss process, the interest rate process and the default intensity process are modeled through the Vasicek model. We derive the closed form formulae for pricing catastrophe options in a reduced form model. Furthermore, we make some numerical analysis on the explicit formulae.

Key words pricing; catastrophe option; counterparty risk; measure change; reduced form model

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1 Introduction

Counterparty credit risk is the risk that the other party will default prior to the expiration of the contract and will not make all the payments required by the contract. Counterparty credit risk is an important topic in the price of credit derivatives in recent years. There are mainly two important models dealing with the price of credit derivatives with counterparty credit risk: the structural model and the reduced form model. The structural model initially proposed in Black and Scholes [1], Merton [2], Black and Cox [3], could give an intuitive understanding. Comparing with the structural model, the reduced form model could give a more flexible model. In recent years, the reduced form model has been studied extensively. For the literature on a reduced form model, we can refer to Lando [4], Jarrow and Yu [5], Seng and Yue [6], Ma and Yun [7], and Su and Wang [8].

Catastrophes are defined as extremely large-scale disasters. There are many severe catastrophes in the past few decades, which had attracted increasing attention to catastrophe derivatives pricing. Cox and Pedersen [9] discussed the price of catastrophe risk bonds through briefly

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discussing the theory of equilibrium pricing and its relationship to the standard arbitrage-free valuation framework. Cox et al. [10] assumed that the asset price process is driven by a geometric Brownian motion with additional downward jumps of a prespecified size in the event of a catastrophe. Jaimungal and Wang [11] supposed that the losses follow a compound Poisson process and the drop in asset price depends on the total loss level, rather than on only the total number of losses. Jiang et al. [12] presented a catastrophe option pricing model that considers counterparty credit risk in a structural model.

Spirited by Ma and Yun [7], Jaimungal and Wang [11], and Jiang et al. [12], we study the price of catastrophe options with counterparty credit risk in a reduced form model. Differently from these papers, we aim at the price of catastrophe options with losses generated by a doubly stochastic Poisson process in a reduced form model. We assume that the share price process is modeled through a jump-diffusion process which is correlated to the loss process, the interest rate process and the default intensity process are modeled through the Vasicek model. We employ measure changes to obtain the closed form formulae for pricing catastrophe options. Furthermore, we make some numerical analysis on the explicit formulae.

The remainder of the paper is organized as follows. In Section 2, we present basic assumptions and provide the necessary preparation for pricing catastrophe options with counterparty credit risk in a reduced form model. In Section 3, we employ several measure changes to derive the closed form formulae for pricing catastrophe options with counterparty credit risk in a reduced form model. We provide numerical analysis in Section 4.

2 The Modeling Assumptions

In this section, we construct a catastrophe put option pricing model with counterparty credit risk in a reduced form model. Specifically, spirited by Jaimungal and Wang [11], we assume that the loss process is generated by a doubly stochastic Poisson process, the share price process is modeled through a jump-diffusion process which is correlated to the loss process, the interest rate process and the default intensity process are modeled through the Vasicek model. About the doubly stochastic Poisson process, we can refer to Lando [4], Jarrow ang Yu [5], Wang and Bi [13], and Liang and Wang [14].

The Vasicek model was proposed by Vasicek [15], which is often applied to the price of derivatives. The disadvantage of this model is that it will lead to the emergence of negative probability. However, the Vasicek model has some very perfect properties, and the model has important reference value for derivatives pricing. So many scholars still use this model for derivatives pricing research, such as Hull and White [16], Shimko et al. [17], Jaimungal and Wang [11].

Consider a filtered probability space $(\Omega, \mathcal{G}, (\mathcal{G}_t)_{t=0}^T, Q)$, where $\mathcal{G} = \mathcal{G}_T$ and Q is an equivalent martingale measure under which the prices of all discounted securities are martingales. Let $\{S(t) : t \geq 0\}$ be the share price process of the issuing company; Let $\{L(t) : t \geq 0\}$ be the loss process of the the issuing company. The dynamics of $S(t)$ and $L(t)$ are given by

$$S(t) = S(0) \exp\{\xi(t) - \alpha L(t) + \beta(t)\}, \tag{2.1}$$