# A numerical optimization technique for design of wheel profiles

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## Abstract

A method for determination of an optimum railway vehicle wheel profile for a given geometric contact characteristics of wheel/rail using a numerical optimization approach is presented in this paper. Such characteristics are defined through a given rail profile, and the target rolling radii difference ($\Delta r$) as a function of lateral displacements ($Y$). Cubic spline functions are utilized to connect curve segments of wheel profile to produce smooth variation trends desired. The coefficients of low degree polynomials could represent the curve's generalized coordinates. In this manner the number of necessary coordinates is greatly reduced. It is shown that the total number of design variables is only five, which computationally makes the optimization process very efficient. The method of Complex algorithm within the non-gradient numerical optimization routines is selected to generate a sequence of improving profiles leading to the optimum one. Dynamic performances of the generated profiles are then determined by obtaining their $Y-\Delta r$ curves using ADAMS/Rail software package. The objective is to minimize the deviation of the $Y-\Delta r$ performances of the generated profiles from the target one, under the given vehicle and track characteristics. Subsequently, the proposed method is used to design a wheel profile for passenger vehicles, compatible with the Iranian north-eastern railway characteristics. The dynamic performances of the designed profile, and those of S1002 and P8 standard profiles, are presented for comparison. It is shown that the wear index of the designed profile is significantly lower than the other two. In addition, other dynamic characteristics such as lateral displacement, angle of attack and derailment quotient are shown to be improved considerably. The procedure worked efficiently and the results proved to be quite satisfactory.
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## 1. Introduction

Problems of wheel/rail interaction have been of interest for many years to rail transportation systems. Several important factors such as wheel/rail wear, dynamic behavior of vehicle and their safety are affected by the geometric shape of wheel/rail profiles, and their contact region characteristics [1–3].

Compatible wheel profiles for given track geometry can provide an adequate rolling radius difference when the wheelset negotiates a curve. As a result, there exist sufficient longitudinal creep forces in contact region of wheel/rail which in turn provide a kind of self steering for rolling stock in curves to some extent [4]. Furthermore, having reduced the amount of lateral creep forces significantly when negotiating a curve, the wear of

wheel flanges would be eliminated in mild curves of radius of 800 m and higher, and would be reduced in rather sharp curves of radius 400 m and less. In addition, to wear elimination or reduction, the mode of deterioration causing the rolling contact fatigue of wheel and rail would be reduced [5–7]. Profiles with conformal contact are expected to provide more compatibility between wheel and rail by providing adequate rolling radius difference (RRD), appropriate equivalent conicity, and preferred distribution of contact points on wheel/rail profiles, resulting in decrease of the mutually tangential contact forces in the contact zones [8–11]. The reduction of wear at the wheel/rail contact results in the reduction of their maintenance, improvement of ride quality, as well as favorably affect the economic aspects of the rolling stock, which are important considerations in a railroad system [10,12].

Despite the recent developments in design of railway vehicles such as inclusion of active steering wheelset, the selection of proper wheel and rail profile is still of much interest. Earlier methods to design these profiles were mainly based on the

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experience of railway operators using various rules of thumb regarding conicity, flange angle, flange root radius, etc. [13]. Yamada et al. [14] and Sasaki et al. [15] used conicity charts to optimize the wheel tread profile in order to improve the curving performance and stability of the rolling stock, and increasing the life of the wheel profile. More recently, optimization methods have been employed to cope with the complexity of wheel profile design. Shevtsov et al. [16] employed a multipoint approximation based on response surface fitting to design an optimum wheel profile that matches a target RRD. The design optimization variables were reduced to 14 moving points. Shen et al. [17] developed a target-oriented method for the design of railway wheel profiles. The design methodology made use of a unique inverse method which employed contact angles and rail profile information. Persson and Iwnicki [18] introduced the genetic algorithm of optimization into the field of wheel profile design.

This paper introduces a numerical optimization method for wheel profile design. The method is a numerical solution to an inverse problem of finding a wheel profile that results in a given $Y - \Delta r$ response for a given track characteristics, similar to the method used in [16]. The use of spline functions for wheel profile approximation has enabled easy integration of the ADAMS/Rail software into the optimization iterations. The low number of design variables and high computational efficiency of the proposed optimization algorithm proved to be quite promising.

The sequence followed to describe the method consists of, problem definition, mathematical modeling, curve generation scheme, and optimization algorithm. Application of the method to wheel profile design of recently constructed north-eastern Iranian railway is presented next. Features of the designed profile including contact points, angle of attack and lateral displacement response with the time, wear index and derailment quotient are compared with those of the known S1002 and P8 widely used standard profiles.

## 2. Wheel/rail geometry and problem definition

Several factors serve to define contact region of wheel and rail known as the geometric constraints. Of these geometric constraints, rolling radius difference and wheel equivalent conicity are considered here as the key factors to define the compatibility of wheel and rail profiles [19,20]. Dynamic behavior of the vehicle is mainly influenced by the geometric constraints of wheel and rail profiles. Traveling on a straight track, the left and right wheels of the rigid wheelset should be aligned and have no difference between their instantaneous rolling radii. However, a rolling radius difference is needed when either traveling on a curve or having lateral excitement on a straight track. In these cases, rolling radius difference versus lateral displacement of the wheel profile plays an important role in the dynamic performance of the vehicle [19]. This rolling radius function depends not only on the wheel and rail profiles but also on factors such as track gauge, rail inclination angle, wheel diameter and unbalanced axle load [16–20].

Different approaches may be used to define the target $Y - \Delta r$. As discussed in [16], irrespective of the case studied, the wheel profile designed on the basis of the target $Y - \Delta r$ should provide suitable stability of the wheelset on a straight track, and at the same time suitable lateral displacement of the wheelset for better curve negotiations and less wear in mild and sharp curves. The procedure to arrive at target $Y - \Delta r$ for the case studied here will be given later in the application section of the paper.

An inverse problem can now be defined. Using the selected $Y - \Delta r$ response as the target, it is conceivable to find the corresponding wheel profile that matches this response for a given rail profile and track geometry. The method presented herein introduces a numerical optimization method with low number of design variables to design a profile for the desired $Y - \Delta r$ curve.

## 3. Mathematical modeling

The inverse problem stated above has the task of producing the required wheel geometry profile which would produce the target rolling radius function for a given set of track data. To do this some key points defining the wheel profile geometry must be chosen. The coordinates of these points would represent the variables defining the analytical form of the profile. Infinite number of curves can be produced by assigning values to these coordinates. In order to arrive at a smooth curve, it is assumed that the curve segments connecting the points are approximated by cubic spline functions. Polynomial functions of low degrees are chosen to represent the various variation trends desired in the profile curve, whose coefficients could represent the curve's generalized coordinates. By so doing, computational efficiency is achieved by lowering the number of necessary coordinates used. This approach, satisfying the physical boundary conditions, and with realistic limits placed on the range of variations of the variables, can virtually produce the collection of curves that can then be utilized in the optimization process.

The mathematical details of the inverse problem are as follows. Eq. (1) below expresses the general format of an optimization problem which has been followed.

$$
\begin{aligned}
&\text{Minimize:} && F(X_1, X_2, ..., X_n) \\
&\text{Subject to:} && G_j(X_i) \leq a_j \text{ and } b_i \leq X_i \leq c_i, \\
& && i = 1, \ldots, n \text{ and } j = 1, \ldots, m
\end{aligned} \tag{1}
$$

In which, $F$ is the objective function, $G_j$ is the $j$th inequality constraint function, $X_i$ is the $i$th design variable, $a_j$ is the right-hand side of $j$th inequality constraint and $b_i$ and $c_i$ are lower and upper limits of design variables, respectively.

The objective function here is the requirement that the rolling radii difference of the optimized wheel profile shows minimum of discrepancy with the target one, defined by Eq. (2) below. The deviation from the target function is taken as a least square function as employed in [16].

$$
\text{Minimize} \rightarrow F = \frac{(\Delta r^{\text{target}} - \Delta r^{\text{optimized}})^2}{(\Delta r^{\text{target}})^2} \tag{2}
$$

In Eq. (2), $\Delta r^{\text{target}}$ and $\Delta r^{\text{optimized}}$ are the target and current rolling radii difference due to the lateral displacement curves,