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Oscillatory dynamics of investment and capacity utilization

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HIGHLIGHTS

- Selected econometric time series are modeled as ordinary differential equations.
- The model predicts relative phases between these variables and their derivatives.
- These predictions have been verified by comparison with US econometric data.

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ABSTRACT

Capitalist economic systems display a wide variety of oscillatory phenomena whose underlying causes are often not well understood. In this paper, I consider a very simple model of the reciprocal interaction between investment, capacity utilization, and their time derivatives. The model, which gives rise periodic oscillations, predicts qualitatively the phase relations between these variables. These predictions are observed to be consistent in a statistical sense with econometric data from the US economy.

1. Introduction

Even a cursory inspection of econometric time series data leaves the sense that many measures show aperiodic oscillatory behavior, that is, alternations of peaks and valleys that give the impression of underlying waves and instabilities. The alternating periods of economic growth and contraction of the business cycle are perhaps the most salient examples. Yet it has proven difficult, if not impossible, to represent adequately this aperiodic oscillatory behavior within the conceptual framework of equilibrium theory, except by recourse to exogenous shocks (e.g. Ref. [1]).

The goal of the work reported here is to identify a very simple dynamical system that is capable of generating oscillatory dynamics, and that is consistent with an empirical understanding of capitalist economic dynamics. The starting point for this exercise is to capture the reciprocal relationship between capacity utilization and investment, and cast this in the form of an ordinary differential equation. The solution to this equation is a pair of time series, one for capital utilization as a function of time, and the other for investment as a function of time.

2. Theory

The intuition for the model is straightforward. First, it seems likely that firms increase their rate of investment when capacity utilization is high, since a higher level of capital investment under conditions of high capacity utilization should

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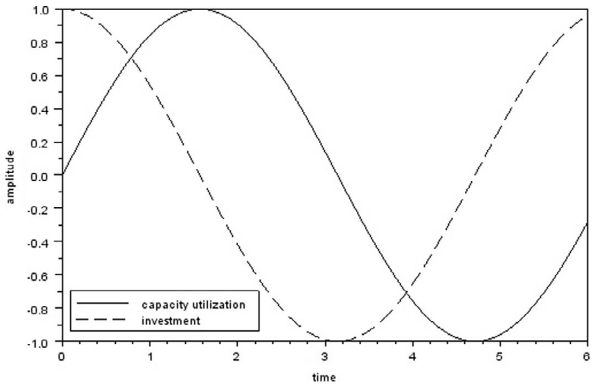

Fig. 1. This figure illustrates a numerical solution to Eq. (3) with parameters $\lambda_1 = \lambda_2 = 1$, and initial conditions $r_I(0) = 1$ and $r_{CU}(0) = 0$. These parameter values are arbitrary and not meant to represent an economic system in any realistic way. They are chosen for convenience to illustrate qualitatively the lag between capital utilization and investment predicted by the simple model. Note that a high level of capacity utilization (e.g., time $= 2$) is correlated after a lag with a low level of investment (e.g., time $= 3$).

lead to increased output, and thus increased profits. Conversely, firms will decrease their rate of investment when capacity utilization is low, since the existing capacity should be adequate for expected future demand, at least in the short term. All things being equal (which, of course, they never are), increased investment will lead to a decrease in capacity utilization, and conversely, decreased investment will lead to increased capacity utilization, thus creating an oscillation.

By hypothesis, the relation between capital utilization and investment has a two-fold character. First I assume that changes in capacity utilization lead to changes in investment. That is, high levels of capacity utilization leads to an increased rate of investment. Second, I assume that high levels of investment lead to a decreased rate of change of capacity utilization. In other words, a positive change in investment leads to a negative change in the capacity utilization level, holding other factors (like demand for consumer goods and overall economic growth) constant.

To convert the preceding qualitative description into symbolic form usable for modeling, write the relations between capacity utilization and investment as a pair of first order ordinary differential equations:

$$\dot{r}_I(t) = \lambda_1 r_{CU}(t) \tag{1}$$

$$\dot{r}_{CU}(t) = -\lambda_2 r_I(t). \tag{2}$$

Eq. (1) expresses the functional relation (a positive correlation) between the time rate of change in investment, $\dot{r}_I(t)$, and the level of capacity utilization, $r_{CU}(t)$, dependent on a free parameter, λ_1. Eq. (2) expresses the functional relation (a negative correlation) between the time rate of change of capacity utilization, $\dot{r}_{CU}(t)$, and the level of investment, $r_I(t)$, dependent on a free parameter, λ_2. Combining these two equations into a single (linear) equation yields

$$\begin{bmatrix} \dot{r}_I \\ \dot{r}_{CU} \end{bmatrix} = \begin{bmatrix} 0 & \lambda_1 \\ -\lambda_2 & 0 \end{bmatrix} \cdot \begin{bmatrix} r_I \\ r_{CU} \end{bmatrix}. \tag{3}$$

The matrix operator clearly has complex eigenvalues, as would be expected for an oscillatory system. For example, if $\lambda_1 = \lambda_2 = 1$, then the eigenvalues are $\pm$i, where $\mathrm{i} = \sqrt{-1}$. In fact, Eq. (3) is isomorphic to the classical harmonic oscillator (e.g., Ref. [2], sec. 6) (see Fig. 1).

Of course, any real economic system cannot possibly be reduced to Eq. (3), for any number of reasons, both trivial (e.g., negative values for r_{CU} are not physically meaningful, economic oscillations are not periodic) and non-trivial (e.g., changes in demand, the influence of the state, financial flows, international trade, depreciation, technological progress, to name only a few). The only justification for such a radical simplification is that it may be useful to isolate a key dynamical feature from the mass of interactions that characterize a real economy, with the hope that such a simplification may lead to both useful insights and testable predictions.

In fact, this simple model makes three testable, semi-quantitative, predictions:

- The model predicts that there should be a positive zero-lag correlation between the level of capacity utilization and the time rate of change of investment, as determined by Eq. (1).
- The model predicts that there should be a negative zero-lag correlation between the level of investment and the time rate of change of capacity utilization, as determined by Eq. (2).
- The model predicts that there should be a time-lagged correlation between the level of investment and the level of capacity utilization, as determined by the solution to Eq. (3).

The correspondences between these predictions and econometric data are described in the Results section and illustrated in Figs. 3–5.