



# Design of arbitrary-order robust iterative learning control based on robust control theory<sup>☆</sup>



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## ABSTRACT

Iterative learning control (ILC) is an effective technique that improves the tracking performance of systems by adjusting the feedforward control signal based on the memory data. The key in ILC is to design learning filters with guaranteed convergence and robustness, which usually involves lots of tuning effort especially in high-order ILC. To facilitate this procedure, this paper proposes a systematic approach to design learning filters for arbitrary-order ILC with guaranteed convergence, robustness and ease of tuning. The filter design problem is transformed into an  $H_\infty$  optimal control problem for a constructed feedback system. This approach is based on an infinite impulse response (IIR) system and conducted directly in iteration-frequency domain. The proposed algorithm is further advanced to the one that explicitly considers system variations based on  $\mu$  synthesis. Important characteristics of the proposed approach such as convergence and robustness are explored and demonstrated through both simulations and experiments on a wafer scanning system.

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## 1. Introduction

Iterative learning control (ILC) is an effective technique to suppress repetitive disturbances and improve the tracking performance of systems that operate in a repetitive manner. It tunes the feedforward control signal iteratively based on the memory data from previous iterations. ILC has been applied to a variety of industrial problems including robot manipulators [1–4], micro positioning stages [5,6], hard disk drives [7,8] and wafer scanning systems [9–11]. Reference [12] provided detailed ILC analysis with applications to various industrial areas.

One main challenge in ILC is to design learning filters to guarantee both the convergence of the tracking error and the robustness to system variations. A common design approach is based on the pseudo-inverse of the plant dynamics, which may be hard to obtain, or introduce a sensitivity problem to unmodeled dynamics [13]. An alternative approach with little tuning effort was proposed based on the  $H_\infty$  optimal control theory [14,15]. This method was further improved by  $\mu$ -synthesis technique to explicitly take

system variations into account with acceptable compromise of the convergence rate [16–19]. Comprehensive reviews of the basic formulations of ILC, its variations and the frequency-domain design approaches were provided in [20–23].

Most research efforts for the  $H_\infty/\mu$ -based approach have focused on first-order ILC which only utilizes the previous iteration. Recently high-order ILC that utilizes more data from previous iterations has gained increasing attention. Compared to first-order ILC, high-order ILC has more flexibilities when designing learning filters and is promising to achieve better performance such as faster tracking or additional robustness to some non-repetitive disturbances [24–28]. Despite such favorable performance, designing multiple learning filters is a difficult task with lots of tuning efforts in high-order ILC. To reduce such efforts, similar to first-order ILC,  $H_\infty$  synthesis was utilized to design learning filters in high-order ILC. For example, in [29,30], the algorithms were proposed in the super-vector framework based on a finite impulse response (FIR) system and the lifting technique, which may result in more computational cost. There is also some research to explore the design of non-causal learning filters for the ILC that assumes finite horizons for each iteration, such as [31,32], which is difficult to extend to the frequency-domain ILC design methodology that assumes infinite horizons for each iteration. To deal with this, Son et al. [33] proposed an approach to take the non-causal case into account and simultaneously optimize the learning filters and the Q filters for first-order ILC. However, the frequency-domain design

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approach for high-order ILC, including both causal and non-causal cases, has not been fully investigated in the existing literature.

This paper develops a systematic frequency-domain design framework for high-order ILC based on the  $H_\infty/\mu$  synthesis to fill in the knowledge gap. In this paper, every iteration is assumed to have infinite horizons, and the systems are represented by infinite impulse response (IIR) filters for easy implementation and efficient computation. The learning filters are generated off line through designing an  $H_\infty$  optimal controller for a constructed feedback system. The ILC algorithms based on  $\mu$ -synthesis are also developed to explicitly consider system variations. The effectiveness of the proposed ILC algorithms are demonstrated on a wafer scanning system through both simulations and experiments. The main contribution of this paper lies in the novel frequency-design approach with systematic inclusion of both first-order and high-order ILCs. This paper extends our previous work on high-order ILC [34], by including the  $\mu$ -based ILC that explicitly considers system uncertainties, and providing detailed design guidelines and experimental validation, as well as preliminary effort in exploring the design of non-causal learning filters using robust control design methodology.

The remainder of the paper is organized as follows. Section 2 reviews the standard ILC formulations and examines the trade-off between the robustness and the repetitive disturbance suppression. Section 3 formulates the design of the learning filter in first-order ILC into an  $H_\infty$  optimization problem. Section 4 extends the formulation in Section 3 by incorporating system variations explicitly based on  $\mu$  synthesis. Then, Section 5 advances the robust formulation of first-order ILC to a general formulation of arbitrary-order ILC with guaranteed convergence and robustness. Section 6 provides the demonstration and validation through simulations and experiments. Section 7 concludes the paper.

## 2. ILC basics

Consider a general discrete-time linear time invariant (LTI) system

$$y = P(u + d) \quad (1)$$

where  $y$  is the output,  $u$  is the control signal,  $d$  is the disturbance, and  $P$  is the plant.  $P$  can be described either by an FIR model:

$$P = h_0 + h_1 z^{-1} + h_2 z^{-2} + \dots \quad (2)$$

or by an IIR model:

$$P = \frac{b_1 z^{-1} + b_2 z^{-2} + \dots + b_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n}} \quad (3)$$

where  $z$  is the discrete frequency domain operator. As mentioned in the introduction, generally the ILC is designed based on the FIR model (2); alternatively, this paper designs the ILC based on the IIR model (3) that may include feedback terms and is more efficient for practical implementation.

The structure of the ILC algorithm for system (1) is shown in Fig. 1, where the reference  $r$  is assumed to remain unchanged through iterations.  $e=r-y$  is the tracking error, and  $u^f$  is the feed-forward control signal that is refined by the ILC algorithm iteration by iteration.  $C$  is a feedback controller.  $u=C(u^f+e)$  is the total real-time control signal. Use  $j$  to index the iterations. By assuming that each iteration is of infinite horizon, the tracking error during the  $j$ th iteration is

$$e_j = T_u u_j^f + T_r r + T_d d_j \quad (4)$$

where  $T_u$ ,  $T_r$ , and  $T_d$  are the closed-loop transfer functions from  $u_j^f$  to  $e$ ,  $r$  to  $e$ , and  $d$  to  $e$ , respectively,

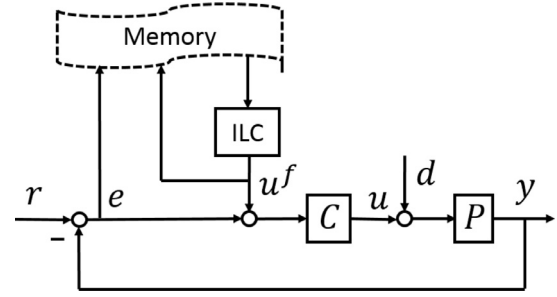


Fig. 1. Control system with ILC.

$$\begin{aligned} T_u &= -(1 + PC)^{-1}PC \\ T_r &= (1 + PC)^{-1} \\ T_d &= -(1 + PC)^{-1}P \end{aligned} \quad (5)$$

A standard first-order ILC is designed as follows,

$$u_{j+1}^f = Q(u_j^f + L e_j) \quad (6)$$

where the filter  $Q$  and the learning filter  $L$  are to be designed. Substituting Eq. (6) into Eq. (4), we have

$$\begin{aligned} e_{j+1} &= T_u [Q(u_j^f + L e_j)] + T_d d_{j+1} + T_r r \\ &= Q(1 + T_u L) e_j + T_r (1 - Q) r + T_d (d_{j+1} - Q d_j) \end{aligned} \quad (7)$$

Assuming that the disturbance  $d$  is consistent through iterations, i.e.,  $d_{j+1} = d_j$ , Eq. (7) becomes

$$e_{j+1} = Q(1 + T_u L) e_j + T_r (1 - Q) r + T_d (1 - Q) d \quad (8)$$

A sufficient condition to guarantee the stability of Eq. (8) with respect to  $e_j$  is

$$\|Q(1 + T_u L)\|_\infty < 1 \quad (9)$$

To eliminate the tracking error, ideally  $Q=1$ , and Eq. (8) becomes

$$e_{j+1} = (1 + T_u L) e_j \quad (10)$$

In this case, if  $\|1 + T_u L\|_\infty < 1$ ,  $e_{j+1}$  converges to zero monotonically over iterations. However, it is usually difficult to find a  $L$  such that  $|(1 + T_u L)(j\omega)| < 1$  is achieved over all frequencies. A major challenge comes from system variations in  $T_u$  which are usually large at high frequencies. Therefore, to obtain robustness against system variations, instead of setting  $Q=1$ ,  $Q$  is often designed as a low-pass filter. In the design of such  $Q$ , the effects on  $T_r(1-Q)r$  and  $T_d(1-Q)d$  must be considered. Nevertheless, the reference  $r$  is usually a low-frequency signal, so that  $T_r(1-Q)r$  is almost zero. The disturbance  $d$ , on the other hand, usually contains high-frequency components, which may enlarge the effect of  $T_d(1-Q)d$  in Eq. (8).

In general, a trade-off exists when designing  $Q$ : the robustness to system variations requires a small gain of  $Q$  at high frequencies, while the disturbance rejection desires a high bandwidth of  $Q$ . To address this trade-off effectively, this paper designs ILC in two steps: (1) design  $L$  through the minimization of  $\|(1 + T_u L)W\|_\infty$ , where  $W$  is the weighting filter to 'shape' the expected frequency response of  $(1 + T_u L)(j\omega)$ ; (2) design  $Q$  to guarantee  $\|Q(1 + T_u L)\|_\infty < 1$ .

## 3. First-order ILC based on $H_\infty$ synthesis

The learning filter (denoted as  $L_\infty$  in this section) design problem can be formulated as an  $H_\infty$  optimization problem:

$$\min_{L_\infty} \|(1 + T_u L_\infty)W\|_\infty \quad (11)$$

where  $W$  is a frequency-dependent weighting filter to provide additional design flexibilities and mitigate the trade-off in the design

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