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Observer-based iterative learning control design in the repetitive process setting ⋆

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Abstract: This paper considers the design of an observer-based iterative learning control law for discrete linear systems using repetitive process stability theory. The resulting design produces a stabilizing feedback controller in the time domain and a PD-type of feedforward controller that guarantees monotonic convergence in the trial-to-trial domain. Furthermore, the new design procedure includes limited frequency range specifications, which will be of particular interest in some applications. All design computations required for the new results in this paper can be completed using linear matrix inequalities. A simulation example is given to illustrate the theoretical developments.

Keywords: Iterative learning control, Linear repetitive processes, Linear matrix inequalities.

Iterative learning control (ILC) arose from research for systems that repeat the same finite duration task over and over again. Each repetition is termed a trial or pass in the literature and the trial (or pass) length is the name given to the finite duration of each trial. The notation for variables used in this paper is $q_k(p)$, $0 \leq p \leq \alpha - 1$, where q is the scalar or vector valued variable under consideration, $k \geq 0$, is the trial number and $\alpha < \infty$ is the number of samples along the trial for discrete dynamics (α times the sampling period gives the trial length).

One each trial is complete, all information generated is available and if stored can be used to compute the control input for the next trial. Hence it is possible to implement ILC laws that use information that would be non-causal in the standard sense, provided it has been generated on a previous trial, e.g., at sample instant p information at $p+\lambda$, $\lambda > 0$, can be used. The inclusion of such information is the distinguishing feature of ILC.

The original work on ILC (Arimoto et al., 1984) considered a derivative, or D type, law for an electric motor and since then this design method has remained as a significant area of control systems research with many algorithms experimentally verified in the research laboratory and applied in industrial applications, see, e.g., (Ahn et al., 2007; Bristow et al., 2006) as starting points for the literature and (Wang et al., 2009) is a starting point for the literature on ILC applications in the chemical process industries. Particular application examples include industrial robotics, see, e.g., (Norrlöf, 2002), where the pick and place operation common in many mass manufacturing processes is an immediate fit, and marine systems, see, e.g., (Sornmo et al., 2016). More recently, ILC algorithms first developed in the engineering domain have been used in robotic-assisted upper limb stroke rehabilitation with supporting clinical trials (Freeman et al., 2009, 2015).

Suppose, in the single-input single-output case with an immediate generalization to the multiple-input multiple-output examples, a reference signal $y_d(p)$ is available for an application. Then the error on trial k is $e_k(p) = y_d(p) - y_k(p)$, $0 \leq p \leq \alpha - 1$, where $y_k(p)$ is the output. The ILC design problem is to construct a control input sequence such that the error sequence converges from trial-to-trial. In formal terms, the requirement is to construct a control sequence $\{u_k\}$ such that

$$\lim_{k\to\infty} ||e_k|| = 0, \ \lim_{k\to\infty} ||u_k - u_\infty|| = 0, \qquad (1)$$

where $||\cdot||$ is a signal norm in a suitably chosen function space with a norm-based topology and u_∞ is termed the learned control.

The finite trial length means that trial-to-trial (in k) error convergence can be achieved even if the system is unstable and hence the presence of 'growth' terms in the transient response (in p) along the trials. One option in such cases is to first design a stabilizing control law and then apply ILC to the resulting controlled system. A commonly used setting for ILC design for discrete dynamics is the lifting setting, which is based on the use of so-called supervectors. Consider again the single-input single-output case. Then since the trial length is finite the sampled values of, e.g., the output can be assembled into a column vector where the entries correspond to their sample instants along the trial. Applying this to all variables enables the error dynamics to be written as a difference equation in k, to which standard results can be applied to design the ILC law.

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An alternative approach to ILC design is to use the two-dimensional(2D)/repetitive systems setting (Rogers et al., 2007, 2015) i.e., systems that propagate information in two independent directions, where for ILC these directions are from trial-to-trial and along each trial respectively. Repetitive processes evolve over a subset of the upper right quadrant of the 2D plane and make repeated sweeps through dynamics defined over a finite duration. Hence, they are a more natural setting for ILC analysis than other 2D systems models. A particular advantage of repetitive process based ILC design is that trial-to-trial error convergence and the dynamics along the trial can be considered in one setting and also the analysis extends to differential dynamics whereas the lifting approach does not.

A common form of ILC law is the proportional plus derivative, or PD-type, consisting of proportional and derivative gains acting on the tracking error. This form of ILC law is one of the earliest developed and as with the three term control laws for standard systems has seen many implementations, see-(Bristow et al., 2006) as a starting point for further details and literature review.

This paper continues the development of the repetitive process setting for ILC design, starting with a new result on the design of PD-type ILC laws where the state feedback control is used. Implementation of such control laws, however, requires the availability for measurement of all state variables. Since this will often not be the case an logical step is to consider the use of a state observer to estimate the current trial state vector entries. The analysis shows that the design problem can be completed by formulation as a stability problem for discrete linear repetitive processes, leading to design based on Linear Matrix Inequality (LMI) computations. An extension to obtain LMI based conditions for ILC design over finite frequency regions, allowing the use of different performance specifications over particular finite frequency ranges, see also Paszke et al. (2016), is developed. This analysis is based on the generalized Kalman-Yakubovich-Popov (KYP) lemma.

Throughout this paper, the null and identity matrices with compatible dimensions are denoted by 0 and I respectively. Moreover, sym(X) is used to denote $X + X^T$ and $X^\perp$ denotes the orthogonal complement. The notation $X \succ Y$ (respectively $X \prec Y$) means that the symmetric matrix $X - Y$ is positive definite (respectively negative definite).The symbol $(\star)$ denotes block entries in symmetric matrices and $\rho(\cdot)$ and $\overline{\sigma}(\cdot)$ denote the spectral radius and maximum singular value, respectively, of their matrix arguments.

Use will also be made of the following results, where the first is the generalized KYP lemma and the second the Elimination (or Projection) Lemma.

Lemma 1. Iwasaki and Hara (2005) Consider matrices $\mathbb{A}$, $\mathbb{B}_0$, Θ and

$$\Phi = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \Psi = \begin{bmatrix} 0 & \mathrm{e}^{\mathrm{j}\omega_c} \\ \mathrm{e}^{-\mathrm{j}\omega_c} & -2\cos(\omega_d) \end{bmatrix}, \tag{2}$$

with $\omega_c = (\omega_l + \omega_u)/2$, $\omega_d = (\omega_u - \omega_l)/2$ and ω_l, ω_u satisfying $-\pi \le \omega_l \le \omega_u \le \pi$. Suppose also that $\det(\mathrm{e}^{\mathrm{j}\omega} I - \mathbb{A}) \neq 0$ for all $\omega \in [\omega_l, \omega_u]$. Then the following statements are equivalent.

i) $\forall \omega \in [\omega_l, \omega_u]$

$$\begin{bmatrix} (\mathrm{e}^{\mathrm{j}\omega} I - \mathbb{A})^{-1}\mathbb{B}_0 \\ I \end{bmatrix}^* \Theta \begin{bmatrix} (\mathrm{e}^{\mathrm{j}\omega} I - \mathbb{A})^{-1}\mathbb{B}_0 \\ I \end{bmatrix} \prec 0. \tag{3}$$

ii) There exist $\mathcal{Q} \succ 0$ and a symmetric $\mathcal{P}$ such that

$$\begin{bmatrix} \mathbb{A} & \mathbb{B}_0 \\ I & 0 \end{bmatrix}^\top (\Phi \otimes \mathcal{P} + \Psi \otimes \mathcal{Q}) \begin{bmatrix} \mathbb{A} & \mathbb{B}_0 \\ I & 0 \end{bmatrix} + \Theta \prec 0. \tag{4}$$

Lemma 2. Gahinet and Apkarian (1994) Given a symmetric matrix $\Gamma \in \mathbb{R}^{p \times p}$ and two matrices Λ, Σ of column dimension p, there exists a matrix $\mathcal{W}$ such that the following inequality holds

$$\Gamma + \mathrm{sym}\{\Lambda^\top \mathcal{W} \Sigma\} \prec 0, \tag{5}$$

if, and only if the following two projection inequalities with respect to $\mathcal{W}$ are satisfied

$$\Lambda^{\perp\top} \Gamma \Lambda^{\perp} \prec 0,\ \Sigma^{\perp\top} \Gamma \Sigma^{\perp} \prec 0, \tag{6}$$

where $\Lambda^\perp$ and $\Sigma^\perp$ are arbitrary matrices whose columns form a basis of the null spaces of Λ and Σ respectively.

Next the required background on discrete linear repetitive processes is given.

1. DISCRETE LINEAR REPETITIVE PROCESSES

The state-space model of a discrete linear repetitive process has the following form (Rogers et al., 2007) over $0 \le p \le \alpha - 1$, $k \ge 0$

$$\begin{aligned} x_{k+1}(p+1) &= \mathcal{A} x_{k+1}(p) + \mathcal{B} u_{k+1}(p) + \mathcal{B}_0 y_k(p), \\ y_{k+1}(p) &= \mathcal{C} x_{k+1}(p) + \mathcal{D} u_{k+1}(p) + \mathcal{D}_0 y_k(p), \end{aligned} \tag{7}$$

where $\alpha < +\infty$ denotes the pass length and on pass k, $x_k(p) \in \mathbb{R}^n$ is the state vector, $y_k(p) \in \mathbb{R}^m$ is the pass profile (output) vector and $u_k(p) \in \mathbb{R}^l$ is the control input vector. The terms $\mathcal{B}_0 y_k(p)$ and $\mathcal{D}_0 y_k(p)$ represent the contribution of the previous pass profile to the current pass state and pass profile vectors respectively.

To complete the process description, it is necessary to specify the boundary conditions, i.e., the state initial vector on each pass and the initial pass profile (i.e., on pass 0). For the purposes of this paper, it is assumed that the state initial vector at the start of each new pass is of the form $x_{k+1}(0) = d_{k+1}$, $k \ge 0$, where the $n \times 1$ vector d_{k+1} has known constant entries. Also it is assumed that the entries in initial pass profile vector $y_0(p)$ are known functions of p over the pass length.

Let $\{y_k\}$ denote the pass profile sequence generated by a repetitive process. Then the unique control problem is that this pass profile sequence can contain oscillations that increase in amplitude in the pass-to-pass direction (k). Hence the stability theory for linear repetitive processes Rogers et al. (2007) requires that a bounded initial pass profile produces a bounded sequence of pass profiles $\{y_k\}$, where the bounded is defined in terms of the norm on the underlying function space.

This stability property can be enforced over the finite pass length of an example or uniformly, i.e., for all possible values of the pass length. The former property is termed asymptotic stability and the latter stability along the pass. The finite pass length means that an example can be asymptotically stable but produce dynamics with unacceptable dynamics along the pass and hence it is stability