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Modeling and Iterative Learning Control of a Circular Deformable Mirror ⋆

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Abstract: An unconditionally stable finite difference scheme for systems whose dynamics are described by a fourth-order partial differential equation is developed using a regular hexagonal grid. The scheme is motivated by the well-known *Crank-Nicolson* discretization that was originally developed for second-order systems and it is used in this paper to develop a discrete in time and space model of a deformable mirror as a basis for control law design. As one example, the resulting model is used for iterative learning control law design and supporting numerical simulations are given.

Keywords: Iterative learning control; Control of partial differential equations; Stability of distributed parameter systems; Linear systems; N-dimensional systems.

1. INTRODUCTION

Discretization of partial differential equations (PDE) describing systems with spatial and temporal dynamics is required to obtain discrete models that can form a basis for the design and digital implementation of control laws. A critical factor for this general approach is numerical stability, i.e., the discrete approximation must produce trajectories close to those produced by the PDE with identical stability properties. One group of methods which can be applied to the discretization of PDEs are based on a finite difference approximation (Strikwerda, 1989).

Discretization of PDEs describing systems or processes with one temporal and one spatial variable, such as the one-dimensional heat transfer equation, results in models that are very similar to repetitive processes (Rogers et al., 2007). The unique characteristic of a repetitive process is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length. In particular, a pass is completed and then the process is reset to the starting location and the next pass can begin, either immediately after the resetting is complete or after a further period of time has elapsed. On each pass, an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. Repetitive processes are therefore a particular case of 2D systems where there are two independent directions of the information propagation.

In the repetitive process representation of the discretization of PDEs, the pass number is associated with the discrete time sample instants and the along the pass dynamics are governed by the discrete spatial variable, see, e.g., (Cichy et al., 2011). One class of the finite difference discretization schemes currently available are those known as explicit (Rabenstein and Steffen, 2011), which were used by (Cichy et al., 2011). These methods produce a causal in time discrete recursive model where at any instant on the current pass a window of sample instants on the previous pass contributes to the dynamics. Such models are known as wave discrete linear repetitive processes and include the extensively studied standard discrete linear repetitive processes as a special case, i.e., when the previous pass contribution at time instant p on the current pass only comes from the same instant on the previous pass.

Explicit discretization methods are conditionally numerically stable, i.e., the time discretization period is related to its spatial counterpart, which leads to the need to use

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dense time and spatial discretization grids. One way of overcoming this drawback is to use the so-called singular methods, see (Rabenstein and Steffen, 2011, 2009) and, in particular, the Crank-Nicolson method (Crank and Nicolson, 1947), which frequently produces an unconditionally stable discrete approximation to the dynamics of the original PDE. Hence, the temporal and spatial grids become independent and can therefore be less dense. However, the resulting discrete model is in implicit form, i.e., there is no straightforward dependence of the pass profile at any instance on the current pass and the window of previous pass values. Instead, this dependence is between windows of data points on the current and previous passes.

The simplest way of formulating and solving control problems for singular systems of the considered class requires the use of the lifting approach, i.e., absorbing the spatial structure of the system into possibly high dimensional vectors, see, e.g., (Cichy et al., 2012) for a detailed treatment. In this paper, the Crank-Nicolson method is extended to systems described by PDEs defined over time and two space variables. As a particular example, a thin flexible plate is considered, which, e.g., can be used to model the vibrations of a deformable mirror subject to a transverse external force. In contrast to (Augusta et al., 2015), a circular plate is considered and a regular hexagonal grid is used for discretization.

Previous results (Augusta et al., 2016) can be applied to show that the resulting discrete approximation has the unconditional numerical stability property and hence, relative to the discrete approximations discussed above, a significantly less dense discretization grid can be used with negligible degradation of the approximate model dynamics. This, in turn, means a much smaller number of sensors and actuators distributed over a controlled plate can be used with advantages in terms of control law design and implementation. As one possible option given a discrete model, an iterative learning control law is designed to achieve a given spatial/temporal reference signal. Supporting numerical simulations are given.

2. PARTIAL DIFFERENTIAL EQUATION REPRESENTATION

The dynamics of the continuous deformable mirror considered in this work are modeled by the following Lagrangian PDE

$$\frac{\partial^4 w(t,x,y)}{\partial x^4} + 2\,\frac{\partial^4 w(t,x,y)}{\partial x^2\,\partial y^2} + \frac{\partial^4 w(t,x,y)}{\partial y^4} + \frac{\rho}{D}\,\frac{\partial^2 w(t,x,y)}{\partial t^2} = \frac{f(t,x,y)}{D}, \tag{1}$$

where

w is the lateral deflection in the z direction [m],
ρ is the mass per unit area [$\mathrm{kg\,m^{-2}}$],
f is the transverse external force, with dimension of force per unit area [$\mathrm{N\,m^{-2}}$],
$\dfrac{\partial^2 w}{\partial t^2}$ is the acceleration in the z direction [$\mathrm{m\,s^{-2}}$],
$D = E\,h^3/(12\,(1-\nu^2))$,
ν is Poisson ratio,
h is the thickness of the plate [m],
E is Young modulus [$\mathrm{N\,m^{-2}}$].

Boundary conditions for the case considered here are discussed in the example section, see (40). Further background on (1) can be found in, e.g., (Timoshenko and Woinowski-Krieger, 1959). Also control action based on an array of actuators and sensors is considered. The sensors are distributed over the entire surface of the plate of the diameter a, but the actuators are only used on the central part of plate with diameter $d < a$. The load hence can be modeled with a Heaviside function H as

$$f(t,x,y) = \big(1 - H(x^2+y^2-d^2)\big)\; q(t,x,y).$$

Since the function $1-H(x^2+y^2-d^2) = 1$ within the region where the load is applied, the distributed system input is set to $f(t,x,y) = q(t,x,y)$ in area of the plate defined by the diameter d and to $f(t,x,y) = 0$ outside this area.

To derive a model suitable for control design, the use of an actuator array requires the discretization of (1) in the spatial variables. Moreover, since the control will be implemented digitally, (1) must also be discretized with respect to time. This task is considered next.

3. DISCRETIZATION AND MODELING

The discretization of (1) is based on finite difference methods, where, in general terms, the following steps must be implemented:

(1) cover the region where a solution is sought by a regular grid, i.e., a regular mesh of nodal points,
(2) replace the derivative terms in the PDE by differences using only values at nodal points, i.e., approximate the derivatives.

To complete these tasks, an implicit discretization of the Crank-Nicolson form will be used. Such a discretization results in an unconditionally numerically stable approximation of the system dynamics, see (Augusta et al., 2016) for a full treatment.

Let p, l, m denote the time instant t_p and the coordinates of nodal points x_l, y_m, respectively. Consider a circular deformable mirror using a regular hexagonal grid. Also let the number of nodal points on the plate diagonal be an odd number denoted by n, i.e., $N = \frac{3\,n^2+1}{4}$ nodal points.

In the discretization method used, derivatives arising in (1) are replaced by finite differences as follows

$$\begin{aligned}\frac{\partial^4 w}{\partial x^4} \approx \frac{1}{4\,\delta_x^4}\big(& w_{p+2,l+2,m} - 2\,w_{p+2,l+1,m+1} - 2\,w_{p+2,l+1,m-1} \\ &+ 6\,w_{p+2,l,m} - 2\,w_{p+2,l-1,m+1} - 2\,w_{p+2,l-1,m+1} \\ &+ w_{p+2,l-2,m} + 2\,w_{p+1,l+2,m} - 4\,w_{p+1,l+1,m+1} \\ &- 4\,w_{p+1,l+1,m-1} + 12\,w_{p+1,l,m} - 4\,w_{p+1,l-1,m+1} \\ &- 4\,w_{p+1,l-1,m+1} + 2\,w_{p+1,l-2,m} + w_{p,l+2,m} \\ &- 2\,w_{p,l+1,m+1} - 2\,w_{p,l+1,m-1} + 6\,w_{p,l,m} \\ &- 2\,w_{p,l-1,m+1} - 2\,w_{p,l-1,m+1} + w_{p,l-2,m}\big),\end{aligned} \tag{2}$$