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Constrained optimal iterative learning control with mixed-norm cost functions☆

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ABSTRACT

Iterative learning Control (ILC) is a widely used design technique for determining feedforward control inputs to systems that perform the same task repeatedly. This feedforward control design problem can be posed as an optimization problem in the Norm Optimal ILC (NO-ILC) framework. Although NO-ILC problems are usually formulated without constraints, they may be extended to enforce constraints by posing an analogous constrained optimization problem, termed Constrained Optimal ILC (CO-ILC). Typical NO-ILC and CO-ILC algorithms use 2-norm type cost functions (i.e., minimizing tracking error and control effort 2-norms), which are smooth and have analytical gradient expressions for design of the update law. However, in many applications, the max (∞) norm of tracking error is critical, which is a nonsmooth function and thus gradient-based ILC methods cannot be directly used. In this manuscript, we design a CO-ILC algorithm to explore the performance of using non-smooth type cost functions and compare them with the case using 2-norm cost function with actuator (or more generally state) box constraints. Specifically, CO-ILC algorithms for linear system with linear constraints are derived using (1) an ∞-norm cost function, (2) a mixed $(2-\infty)$-norm cost function and (3) a sequential $(2-\infty)$-norm cost function; the performance results of these are then compared with the traditional CO-ILC using 2-norm cost function on a precision motion control stage experimentally. We also provide proofs for robust monotone convergence of the proposed CO-ILC algorithms for a class of uncertainty models.

1. Introduction

Iterative learning control (ILC) algorithms improve performance for systems that execute repetitive tasks (typically by minimizing tracking error and control effort) by incorporating error information from past iterations (or trials/passes) into the control signal for the current iteration of the repetitive process. ILC has been successfully used in many applications such as industrial robots [1], motion control systems[2], wet clutch control [3], assistive rehabilitation robotics [4] and quadcopter flight control [5] because of its ability to realize excellent tracking performance in spite of imperfect model knowledge and repetitive disturbances.

Since ILC aims to refine the feedforward control signal from iteration to iteration using error profiles from past iterations, the ILC problem can be recast as an optimization problem being solved in an iterative manner. One of the earliest optimization-based ILC algorithms was proposed in [6], where a quadratic cost function was optimized. Since then, several Norm Optimal ILC (NO-ILC) schemes (which design ILC update laws by minimizing a “next-iteration” cost function) have been explored in several papers [7–10], the robustness and monotonicity properties of this family of algorithms have also been investigated [11,12].

In most practical applications of ILC, there are constraints imposed on control effort, states, rate of change of state and control variables, etc. One approach to address these constraints in NO-ILC is by tuning the cost function (by including a weighting parameter on the control effort, etc.) The NO-ILC framework can be extended to enforce constraints through the formulation of a constrained optimization problem, termed *Constrained Optimal ILC (CO-ILC)*. Recently, several algorithms have been developed for ILC design for systems with constraints. Mishra et al. [13,14] proposed a modified interior-point-type method that combined experimental data in solving the optimization based ILC problem for minimizing tracking error with input saturation. Chu et al. [15] proposed a successive projection framework to solve the CO-ILC problem, while Freeman et al. [16] used an interior-point method to address constraints for the point-to-point ILC tracking problem, with the requirement of point-to-point tracking embedded as an equality constraint. Volckaert et al. [17] added a model correction step and

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then used a sparse implementation of interior-point method to solve the optimal ILC problem for a class of nonlinear systems with constraints. Janssens et al. [18] proposed a method to estimate a linear time invariant system's impulse response using previous iterations input/output data and then solved the constrained optimization problem using a commercial solver such as CPLEX.

These algorithms have all been successfully demonstrated for solving the constrained ILC problem with an 2-norm type cost function. This norm is widely used as the cost function because of its smoothness (and thus the existence of a unique and bounded gradient/derivative). However, many applications require the optimization of non-smooth cost functions. For example, in trajectory tracking applications (such as disk drive control), we may wish to reduce the peak tracking error, which is the ∞-norm of error. In active suspension control [19], we want to minimize the magnitude of the acceleration and the displacement of a car body, thus the ∞-norms of acceleration and displacement are also important here. Therefore, a generalized framework for addressing a larger set of cost functions including non-smooth type cost functions can be very beneficial for many applications. Schoellig [20] explored the convergence performance of using 1, 2 and ∞ norm in the cost functions for a quadrotor reference tracking problem, in which the previous iterations' information is used to estimate the disturbance (in some norm sense) and then the input is updated by solving a constrained quadratic program (QP) (as a one-shot optimization problem). Further, they also showed that error and control effort do indeed converge while using these norms in the objective function. In the author's previous paper [21], a framework for CO-ILC using different norms (1, 2 and ∞) in cost functions was proposed. Instead of directly solving the constructed constrained QP or linear program (LP) in one-shot, a modified interior-point method is used to solve the problem recursively, and the previous iteration's measurements were used directly in every step of the optimization algorithm. The converged error profiles of CO-ILC using 2-norm, ∞-norm and mixed-norm in cost functions were also compared based on simulation results.

In this paper, the CO-ILC framework proposed in [21] is developed further, a sequential $(2-\infty)$-norm optimization problem is posed and solved. A proof for robust monotone convergence of the proposed algorithms for a class of uncertainties is also provided. The main contributions of this paper are the formulation of CO-ILC problems with non-smooth type cost functions (specifically ∞-norm, mixed $(2-\infty)$-norm and a sequential $(2-\infty)$-norm cost function), the proof of the proposed algorithm's robustness, and experimental validation and comparison of the proposed algorithms on a linear motion stage, which are not addressed in [13,21].

The remainder of the paper is organized in the following manner: Section 2 describes the system and the general form of optimization problems considered as QPs/LPs, then an interior-point-type ILC update law is introduced to solve this class of problems. Section 3 presents the formulation of CO-ILC with non-smooth cost functions as a constrained QP or LP, the proof for robust monotone convergence under certain uncertainty is also presented. Then, the experimental results of implementing the proposed algorithms on a linear motion stage will be presented in Section 4. Finally, conclusions and open issues to be addressed in the future are presented in Section 5.

2. Constrained optimal ILC

In this section, we formulate the CO-ILC problem for a linear time invariant discrete-time system (stabilized in closed-loop) and present a modified interior-point-type method as the learning law. While we have developed the problem formulation with actuator saturation as a typical constraint, it is straightforward to extend

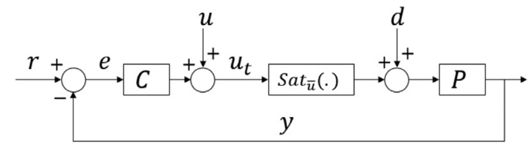

Fig. 1. Block diagram of the closed-loop system with actuator saturation at the plant input. The feedforward input u is to be designed through ILC.

this idea to (as will be done in Section 3) an arbitrary linear constraint on the design variable (in this case, the feedforward control effort). Therefore, state constraints (for linear systems) can also be incorporated into this framework.

2.1. System description

Consider the stable single input single output[1] (SISO) closed-loop system with input saturation shown in Fig. 1. A process (i.e., a trajectory) is executed repeatedly by this system starting at rest condition for each iteration. Let P represent the discrete linear time-invariant (LTI) plant, which is stabilized by an LTI feedback controller C. Then $y,\ u,\ u_t,\ e \in \mathcal{R}$ respectively denote the output, the feedforward control effort, the total control effort and the error. Moreover, $r, d \in \mathcal{R}$ indicate the output reference trajectory and repetitive disturbance respectively. The saturation constraint clips the control effort u_t to $|u_t| \le \bar{u}$, where $\bar{u}$ is the maximum admissible total control effort. Assuming that the input saturation constraint is never violated (i.e., we always operate in the linear regime), system output, total control effort and error are:

$$\begin{aligned} y &= g_{ry}(z^{-1})r + g_{uy}(z^{-1})u + g_{dy}(z^{-1})d \\ u_t &= g_{ru}(z^{-1})r + g_{uu}(z^{-1})u + g_{du}(z^{-1})d \\ e &= r - y, \end{aligned} \tag{1}$$

where g_{ry}, g_{uy}, and g_{dy} are transfer functions from r, u, and d to y respectively, g_{ru}, g_{uu}, and g_{du} are transfer functions from r, u, and d to u_t, z^{-1} denotes the unit delay. The repetitive nature of the process makes this system a two-dimensional system; with evolution along an iteration (time) and from iteration to iteration [22].

In order to construct the ILC design as an optimization problem, in this manuscript we use the lifted system description of this system to transform it into a one-dimensional system along the iteration axis only. Lifting the system [23] yields:

$$\begin{aligned} \mathbf{y}_k &= \mathbf{G}_{ry}\mathbf{r} + \mathbf{G}_{uy}\mathbf{u}_k + \mathbf{G}_{dy}\mathbf{d} \\ \mathbf{u}_{t,k} &= \mathbf{G}_{ru}\mathbf{r} + \mathbf{G}_{uu}\mathbf{u}_k + \mathbf{G}_{du}\mathbf{d} \\ \mathbf{e}_k &= \mathbf{r} - \mathbf{y}_k, \end{aligned} \tag{2}$$

where $\mathbf{r}$, $\mathbf{d}$, $\mathbf{y}_k$, $\mathbf{u}_k$, $\mathbf{u}_{t,k}$, $\mathbf{e}_k$ are the vectors that stack up all the corresponding signals in the kth iteration, for example, $\mathbf{u}_k = [\mathbf{u}_k(0), \mathbf{u}_k(1), \ldots, \mathbf{u}_k(N-1)]^T$, N is the number of time samples in one iteration. The matrices $\mathbf{G}_{ry}$, $\mathbf{G}_{dy}$, $\mathbf{G}_{uy}$, $\mathbf{G}_{ru}$, $\mathbf{G}_{du}$, $\mathbf{G}_{uu} \in \mathcal{R}^{N\times N}$ are lower triangular Toeplitz matrices determined from the impulse response coefficients of the corresponding transfer functions.

2.2. Constrained ILC design as an optimization problem

Designing an ILC algorithm implies determining a learning function that incorporates information of the previous iteration(e.g., feedforward control signal and the corresponding error...) into the generation of the feedforward control signal for the next iteration. We can express this generally as $\mathbf{u}_{k+1} = F(\mathbf{u}_k, \mathbf{e}_k, \ldots)$, where F is the learning function.

[1] The development here is limited to the SISO case only for brevity of notation. With suitable modifications to the notation, the algorithm for a multiple-input multiple-output (MIMO) system can also be developed analogously.