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Higher-order Iterative Learning Control Law Design using Linear Repetitive Process Theory: Convergence and Robustness

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Abstract: Iterative learning control has been developed for processes or systems that complete the same finite duration task over and over again. The mode of operation is that after each execution is complete the system resets to the starting location, the next execution is completed and so on. Each execution is known as a trial and its duration is termed the trial length. Once each trial is complete the information generated is available for use in computing the control input for the next trial. This paper uses the repetitive process setting to develop new results on the design of higher-order ILC control laws for discrete dynamics. The new results include conditions that guarantee error convergence and design in the presence of model uncertainty.

Keywords: iterative learning control, higher-order laws, 2D systems.

1. INTRODUCTION

The first research on Iterative Learning Control (ILC) developed a derivative, or D-type, law for speed control of a voltage-controlled dc-servomotor. Since this first work ILC has been an established area of research and one starting point for the literature is the survey papers Ahn et al. (2007); Bristow et al. (2006). A large volume of the currently available literature assumes a discrete model of the dynamics is available, by sampling if required and hence direct digital design.

The novel feature of ILC is that all information generated on previous trials is known and can be used in the control law. In higher-order ILC, there are contributions from a finite number $M > 1$ of previous trials. The notation for variables in this paper is of the form $h_k(p), 0 \leq p \leq \alpha - 1$, where h is the vector or scalar valued variable of interest, the integer $k \geq 0$ is the trial number and α is the number of samples along the trial (α times the sampling period gives the trial length).

Suppose that $r(p), 0 \leq p \leq \alpha - 1$, denotes the supplied reference vector or signal. The error on trial k is $e_k = r(p) - y_k(p)$, where $y_k(p)$ is the output on this trial. Then the ILC design problem is to construct a control sequence $\{u_k\}$ such that the error sequence $\{e_k\}$ converges, i.e., sequentially improve the tracking error from trial-to-trial. Moreover, once a trial is complete all information generated on this trial is available for use in computing the control signal for the next trial and hence non-causal terms are allowed in the current trial input. Higher-order ILC (Bien and Huh, 1989), where information from the previous $M > 1$ previous trials is used in the computation of the current trial control input, offers the possibility of a higher error convergence speed compared to the standard form of an ILC law where only previous trial information is used.

Given the finite trial length, one approach to ILC design for discrete systems is to represent the dynamics by an equivalent standard systems model, where, e.g., the trial output is represented by a column vector formed from the values at the sample instants along the trial. This is often termed lifted ILC design and given the reference trajectory, the trial-to-trial error dynamics can be written as a discrete difference equation in the trial number. The basic task then is to design the ILC law such that trial-to-trial error convergence occurs. In this design setting it is assumed that the system is stable but if not a preliminary feedback control must be applied to ensure stability and acceptable transient dynamics along the trials and ILC designed for the resulting controlled dynamics.

Lifted ILC design is a two-stage procedure and an alternative is to exploit 2D systems theory where in this setting one indeterminate is the trial number and the other the along the trial variable. Repetitive processes are a distinct class of 2D systems where information propagation in one direction only occurs over a finite duration this is an inherent property of the dynamics and not an assumption introduced for analysis purposes. A detailed treatment of repetitive processes, including industrial examples and how this setting can be found in Rogers et al. (2007).

As the trial length is finite, repetitive processes are a closer match to ILC and designs using this setting have seen experimental verification on a gantry robot that replicates the pick and place operation, see, e.g. Hladowski et al. (2010); Paszke et al. (2016). In the repetitive process setting, it is possible to do control law design for error convergence and transient dynamics along the trials in one step. Moreover, unlike the lifted approach, this setting extends naturally to differential dynamics, i.e., to cases

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where design by emulation is the preferred or only setting for design.

Robustness, as in other areas, is an important issue in ILC design. In standard linear systems theory one commonly used setting for robustness and control law design is to assume that the uncertainty present lies in a specified model class. Two commonly used classes are termed norm-bounded and polytopic, respectively and in this paper the former is considered in the ILC setting. The result is LMI-based control law design algorithms that extend naturally to the polytopic case.

In recent work (Wang et al., 2016) it was shown that higher-order ILC laws can be developed using linear repetitive process theory. The performance of the resulting design was illustrated using numerical simulations. These previous results show that higher-order ILC law is able to achieve tracking error convergence, but the nature of the convergence was unanswered and, moreover, a rigorous proof of the convergence properties was also missing. In this paper, this previous work is extended by the development of a novel higher-order ILC design with guarantee error convergence, which, in fact, has a form of monotonicity (trial-to-trial) convergence. A rigorous convergence proof is developed and it is shown that design in the presence of uncertainty can also be undertaken in the repetitive process setting.

As an essential step to experimental validation, the design algorithms developed in this paper are applied to a model of a physical system. This is a model for one axis of a gantry robot executing a pick and place operation, to which ILC is particularly suited. The model used was obtained by frequency response tests on the robot, which has been used to experimentally verify a number of ILC laws, see, e.g. Hladowski et al. (2010); Paszke et al. (2016).

Throughout this paper, the null and identity matrices with the required dimensions are denoted by 0 and I respectively. Also $M \succ 0$ ($\prec 0$) denotes a real symmetric positive (negative) definite matrix and $X \preceq Y$ is used to denote the case when $X - Y$ is a negative semi-definite matrix. Finally, the symbol $\{*\}$ denotes block entries in a symmetric matrix and $\rho(\cdot)$ the spectral radius of its matrix argument, i.e., if a square matrix, say H, has eigenvalues h_i, $1 \leq i \leq j$, $\rho(H) = \max_{1\leq i\leq j} |h_i|$.

2. BACKGROUND

Consider the discrete linear time-invariant state-space model described in the ILC setting by

$$\begin{aligned} x_k(p+1) &= Ax_k(p) + Bu_k(p), \\ y_k(p) &= Cx_k(p),\ p = 0, 1, ..., \alpha - 1, \end{aligned} \tag{1}$$

where $k \geq 0$ is the trial number, $\alpha < \infty$ is the finite number of samples along the trial, i.e., α times the sampling period is equal to the trial length. Also on trial k $x_k(p) \in R^m$ is the state vector, $y_k(p) \in R^n$ is the output vector and $u_k(p) \in R^s$ is the input vector. Let $r(p) \in R^n$ denote the reference vector and hence the error on kth trial is

$$e_k(p) = r(p) - y_k(p). \tag{2}$$

The ILC design requirement of forcing the error sequence $\{e_k\}$ to converge in k can be formulated mathematically as

$$\lim_{k\to\infty} \|e_k\| = 0, \quad \lim_{k\to\infty} \|u_k - u_\infty\| = 0, \tag{3}$$

where u_∞ is termed the learned control and $\|\cdot\|$ denotes the norm on the underlying function space. One class of widely considered ILC laws computes the current trial input as the sum of that used on the previous trial plus a correction term computed using previous trial data, i.e.,

$$u_{k+1}(p) = u_k(p) + \Delta u_{k+1}(p), \tag{4}$$

where $\Delta u_{k+1}(p)$ is the correction to be designed.

For analysis purposes only, define the following vector from (1)

$$\eta_{k+1}(p+1) = x_{k+1}(p) - x_k(p), \tag{5}$$

i.e., the difference between the state vectors on successive trials. Also consider the case when

$$\Delta u_{k+1}(p) = K_1\eta_{k+1}(p+1) + K_2 e_k(p+1), \tag{6}$$

where K_1 and K_2 are compatibly dimensioned matrices to be designed. Combing (1), (4) and (6), gives

$$\begin{aligned} \eta_{k+1}(p+1) &= \hat{A}\eta_{k+1}(p) + \hat{B}e_k(p), \\ e_{k+1}(p) &= \hat{C}\eta_{k+1}(p) + \hat{D}e_k(p), \end{aligned} \tag{7}$$

where

$$\begin{aligned} &\hat{A} = A + BK_1,\ \hat{B} = BK_2, \\ &\hat{C} = -C(A + BK_1),\ \hat{D} = I - CBK_2, \end{aligned} \tag{8}$$

Repetitive processes make a series of passes through a set of dynamics defined over a finite duration known as the pass length. Once each pass is complete, the process resets to the starting location and the next pass can begin, either immediately after the resetting operation is complete or after a further period of time has elapsed. On each pass an output, termed the pass profile, is produced, which acts as a forcing function on, and hence contributes to, the dynamics of the next pass. A detailed treatment of the dynamics and control problems for linear repetitive processes can be found in Rogers et al. (2007). The ILC dynamics (6) are those of a discrete linear repetitive process with previous pass profile $e_k(p)$, current pass state vector $\eta_{k+1}(p)$ and no control input. From this point onwards, pass is replaced by trial to conform with the majority of the ILC literature.

The stability theory for linear repetitive processes (Rogers et al., 2007) requires that a bounded initial trial profile vector produces a bounded sequence of trial profile vectors, where boundedness is defined in terms of the norm on the underlying function space. This theory is based on an abstract model in a Banach space setting that includes all constant trial length linear examples as special cases.

Two forms of stability are defined, termed asymptotic and along the trial, respectively, where the former imposes the boundedness property over the finite and fixed trial length and the latter is stronger since it requires this property for all possible values of the trial length. Moreover, this latter property can be analyzed mathematically by considering $\alpha \to \infty$.

Theorem 1. (Rogers et al., 2007) The state-space model (7) describing the ILC dynamics is stable along the trial if and only if

- $\rho(\hat{D}) < 1$,
- $\rho(\hat{A}) < 1$,