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Nonlinear behaviour of cantilevered carbon nanotube resonators based on a new nonlinear electrostatic load model

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ABSTRACT

The present study examines the nonlinear behaviour of a cantilevered carbon nanotube (CNT) resonator and its mass detection sensitivity, employing a new nonlinear electrostatic load model. More specifically, a 3D finite element model is developed in order to obtain the electrostatic load distribution on cantilevered CNT resonators. A new nonlinear electrostatic load model is then proposed accounting for the end effects due to finite length. Additionally, a new nonlinear size-dependent continuum model is developed for the cantilevered CNT resonator, employing the modified couple stress theory (to account for size-effects) together with the Kelvin-Voigt model (to account for nonlinear damping); the size-dependent model takes into account all sources of nonlinearity, i.e. geometrical and inertial nonlinearities as well as nonlinearities associated with damping, small-scale, and electrostatic load. The nonlinear equation of motion of the cantilevered CNT resonator is obtained based on the new models developed for the CNT resonator and the electrostatic load. The Galerkin method is then applied to the nonlinear equation of motion, resulting in a set of nonlinear ordinary differential equations, consisting of geometrical, inertial, electrical, damping, and size-dependent nonlinear terms. This high-dimensional nonlinear discretized model is solved numerically utilizing the pseudo-arclength continuation technique. The nonlinear static and dynamic responses of the system are examined for various cases, investigating the effect of DC and AC voltages, length-scale parameter, nonlinear damping, and electrostatic load. Moreover, the mass detection sensitivity of the system is examined for possible application of the CNT resonator as a nanosensor.

1. Introduction

Nanoelectromechanical systems (NEMS) are becoming more common as new fabrication technologies emerge. These nanoscale devices have very high quality factors [1–3], significantly high operating frequency, and low power consumption; such features allow NEMS devices to be implemented as nanoresonators and nanosensors [4] for ultrasensitive mass and bio-sensing applications [5]. Having the desired electrical and mechanical properties, carbon nanotubes (CNTs) [6] are considered suitable to be used as nanoscale resonators, as they have been successfully implemented in NEMS devices [7].

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1.1. Literature review

A large amount of research has been carried out on developing models for carbon nanotube resonators and examining the possibility of using such devices as mass sensors. Most of the studies in the literature are based on continuum models. For instance, Poot et al. [8] investigated the transverse oscillation in suspended carbon nanotubes and analyzed the electrostatic force and the gate tuning. Li and Chou [5] analyzed the mass-detection sensitivity of cantilevereded and clamped-clamped CNT resonators and obtained a logarithmic linear relationship between the resonant frequency and the attached mass. Wu et al. [9] contributed to the field by examining the resonant frequency and mode shapes of a cantilevereded single-walled carbon nanotube analytically and numerically; they also studied the resonant frequency shift of a CNT caused by the addition of a particle to the tip. Ouakad and Younis [10,11] investigated the linear and nonlinear dynamic response of doubly clamped and cantilevereded CNT resonators employing an eigenvalue analysis and the method of multiple scales, respectively.

Studies on this topic were continued by Hajnayeb and Khadem [12], who developed a model for nonlinear oscillations of a double-walled clamped-clamped CNT resonator; they used a Taylor series approximation for the electrostatic load and employed the method of multiple scales, along with a single-mode approximation, to examine the system behaviour. Further investigations were conducted by Mei and Li [13], who developed a model for a doubly clamped carbon nanotube resonator to examine the tunability of the resonant frequency under thermal load for adjusting the mass detection sensitivity of the resonator. Vignola and Judge [14] analyzed different architectural factors in micro- and nano-resonators for mass detection in different environments, such as air and water. Fakhrabadi et al. [15] studied the static deflection and pull-in characteristics of several carbon nanotubes with different dimensions and boundary conditions, under electrostatic actuation. Souayeh and Kacem [16] examined the nonlinear vibrations of a cantilevereded CNT-based mass sensor, employing a three-degree-of-freedom discretized model, while replacing the electrostatic load with a fifth-order Taylor series approximation; they used the harmonic balance method to solve the discretized model. Further investigations were carried out by Farokhi et al. [17], who examined the nonlinear dynamics of a doubly clamped CNT resonator numerically employing a 16-degree-of-freedom discretized model, while keeping the electrostatic load term intact; they showed that for accurate modelling and reliable predictions, the electrostatic load term needs to be kept intact and *not* to be replaced by a Taylor series approximation.

Furthermore, several experimental investigations have been carried out on the nonlinear damping mechanisms in nano resonators [18]. For instance, Unterreithmeier [19] studied the transverse oscillatory modes of a nanomechanical silicon nitride resonator; they observed amplitude-dependent damping and attributed the nonlinear damping mechanism to the interaction of the strain with local defects. Croy et al. [20] analyzed the mechanisms for dissipation in nanoelectromechanical graphene resonators; they showed that the coupling between flexural modes and in-plane phonons leads to linear and nonlinear damping of transverse vibrations. Moreover, they explained that by tuning external parameters such as the DC and AC voltages, one can cross over from a linear to a nonlinear-damping dominated regime. A thorough experimental investigation on the damping mechanisms in carbon nanotube and graphene resonators was performed by Eichler et al. [21], who showed that the damping in such systems depends strongly on the amplitude of the motion. More specifically, they observed nonlinear damping in two distinct systems, i.e. carbon nanotube resonators and graphene resonators, and also under different conditions, i.e. under tensile stress and slacked, and hence concluded that the damping must be modelled as *nonlinear and amplitude-dependent*. They also mentioned the geometrical nonlinearity as one of the physical origins of the nonlinear damping in such nano resonators; such a nonlinear damping mechanism can be modelled via a viscoelastic model [22] such as Kelvin-Voigt.

All of the valuable studies in the literature, which examined the behaviour of an electrostatically actuated CNT resonator based on a continuum model, used a model for the electrostatic load which is for an *infinitely long* CNT resonator [23], and (as will be discussed in the next section) does not take into account the end effects (i.e. the effects of clamped and free ends, in the case of a cantilevereded CNT resonator) due to finite length of the CNT resonator. Moreover, all of those studies considered a linear viscous damping mechanism to model energy dissipation; however, as explained in the previous paragraph, damping in CNT resonators must be considered nonlinear and amplitude-dependent in order to obtain reliable results.

1.2. Contributions of the present study

In the present study, a 3D finite element model is developed in order to obtain the electrostatic load distribution on cantilevered CNT resonators. A new nonlinear electrostatic load model is then proposed which, similar to the 3D finite element model, accounts for the electrostatic load variation near the boundaries, i.e. it accounts for the clamped and free end-effects due to finite length. A nonlinear size-dependent continuum model is developed for the CNT resonator, based on the modified couple stress theory and the Kelvin-Voigt model; Hamilton's principle is employed to obtain the nonlinear size-dependent equation of motion. This is the first time that for a cantilevered CNT resonator, all sources of nonlinearity, i.e. geometric, inertial, dissipative, and size-dependent ones, are taken into account; it is worth noting that the nonlinear studies in the literature account only for geometric nonlinear terms (i.e. neglect inertial, dissipative, and size-dependent nonlinear terms). The new nonlinear continuous model, consisting of the new nonlinear size-dependent equation of motion together with the new electrostatic load model,[1] is discretized into a high-dimensional set of nonlinear ordinary differential equations employing the Galerkin scheme, while keeping the new electrostatic load term intact (i.e. retaining the nonlinearities in the denominator of the electrostatic load

[1] We use the expression "new electrostatic load model" meaning the new analytical expression for the electrostatic load, as introduced in this paper.

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