A spectral load model for pedestrian excitation including vertical human-structure interaction

Elisa Bassoli[a,*], Katrien Van Nimmen[b,c], Loris Vincenzi[a], Peter Van den Broeck[c]

[a] University of Modena and Reggio Emilia, Department of Engineering "Enzo Ferrari", Modena, Italy
[b] KU Leuven, Department of Civil Engineering, Structural Mechanics Section, Leuven, Belgium
[c] KU Leuven, Department of Civil Engineering, Technology Cluster Construction, Structural Mechanics and Building Materials Section, Technology Campus Ghent, Belgium

ARTICLE INFO

ABSTRACT

This paper is focused on the evaluation of the structural response to vertical pedestrian excitation for a wide range of footbridge and crowd parameters. A spectral load model for pedestrian-induced forces proposed in literature is adopted. The model accounts for the randomness in the human excitation as well as the increased correlation among pedestrians with pedestrian density. Therefore, it can be applied for the vibration serviceability analysis of footbridges in unrestricted and crowded traffic conditions. With the purpose of predicting the structural response for a wide range of natural frequencies, an extension of the model to account for the contribution of the first three harmonics of the walking load is proposed. To allow for a more accurate prediction of the maximum response, the present study in addition accounts for the vertical mechanical interaction between pedestrians and the supporting structure. Finally, the impact of human-structure interaction (HSI) on the structural response is investigated. By applying the methods of linear random dynamics, the maximum dynamic response of the footbridge is evaluated based on an analytical formulation of the load and the frequency response function (FRF) of the coupled crowd-structure system. The most significant HSI-effect is in the increase of the effective damping ratio of the coupled crowd-structure system that leads to a reduction of the structural response. However, in some cases the shift in frequency of the coupled crowd-structure system results into a higher structural response when HSI-effects are accounted for.

1. Introduction

The availability of advanced design methods and high strength materials has made modern footbridges increasingly slender and lively structures, prone to human-induced vibrations. Hence, vibration serviceability has become a topical issue in the design of modern slender footbridges with large spans. Different deterministic time domain models to simulate the pedestrian action have been developed [1–4], some of which are adopted by the current design guidelines for footbridges (e.g. [5,6]). Time domain load models are typically based on the hypothesis that both feet produce exactly the same periodic force [7]. This approach does not adequately address essential features of the walking process, that is in fact stochastic and narrow-band [8]. Brownjohn et al. [9] have analysed real continuous walking forces obtained from an instrumented treadmill and the effect of their random imperfections on the resulting structural response. Their analyses show that there are significant differences between responses due to the imperfect real walking forces and the equivalent perfectly periodic model. Similar results are obtained from other researchers, such as

[10–12].

While the perfectly periodic model, which is expanded in Fourier coefficients with a limited number of harmonics, allows for a deterministic analysis, a stochastic analysis is more suited to account for the random nature of human walking. In the frequency domain this can be addressed by representing the walking forces as equivalent power spectral densities (PSDs) [9]. The introduction of spectral models of pedestrian excitation [8,9,13–15] allows to asses the vibration serviceability in the frequency domain through the methods of linear random dynamics [16]. Furthermore, the walking behaviour is influenced by HSI as well as interaction with other pedestrians on the structure (human–human interaction), which is increasingly relevant for higher densities [17,18]. Some experimental investigations relating to human-human interaction can be found in literature, e.g. [19–21]. They show that the mean step frequency of a group or crowd of pedestrians generally decreases with increasing pedestrian density. Similarly, they observe that the variability of the step frequency among pedestrians in the crowd tends to decrease with increasing pedestrian density. Spectral load models accounts for relevant stochastic

---

* Corresponding author.
E-mail address: elisa.bassoli@unimore.it (E. Bassoli).

parameters of the walking force, e.g. pacing rate, as well as for human-human interaction by accounting for the correlation among pedestrians at different locations on a structure. This latter approach is similar to estimating the dynamic response of structures to turbulent buffeting by wind [9,22].

The frequency domain approach proposed by Brownjohn et al. [9] for the vertical forces induced by groups of normally walking pedestrians accounts for the imperfection of individual walking excitation as well as the statistical distribution of pacing rates in the walking crowd. However, the model can only be adopted to assess the effect of unrestricted pedestrian traffic, as the correlation among pedestrians is not accounted for. Unrestricted pedestrian traffic occurs for low pedestrian densities, i.e. when pedestrians arrive randomly and are able to move undisturbed, each one with his own characteristics in terms of loading amplitude, pacing frequency, velocity and phase.

With the same objective, Krenk [14] formulates a model for the pedestrian load in the form of a stochastic process, based on a rational function representation of the spectral density of the process. Starting from the representation of the pedestrian load via the frequency spectral density, Krenk proposes simplified expressions to evaluate the standard deviation of the dynamic response by introducing an effective damping ratio. The latter is obtained combining the damping ratio of the structure and the bandwidth parameter of the load process. The proposed expressions can be adopted to estimate the dynamic response to both resonant and non-resonant loading, in unrestricted pedestrian traffic conditions.

The present paper focuses on the spectral model proposed by Piccardo, Tubino and Ferrarotti [13,22] for the modal force induced by pedestrian groups. The model allows to take into account the intrinsic randomness of the walking force. It relies on an analytical definition of the PSD of the modal force that can be adopted in unrestricted traffic conditions [13] as well as crowded conditions [22]. In the latter case, a coherence function is introduced, which accounts for the increased synchronization among pedestrians with increasing pedestrian density. The benefit of this method is that, applying the principles of linear random dynamics, the maximum structural response can be evaluated by means of simple closed-form expressions.

In addition, the presence of a crowd of pedestrians can cause significant changes in the dynamic characteristics of the coupled crowd-structure system with respect to the empty footbridge [23–25]. In fact, pedestrians are mechanical systems which interact with the structure that is supporting them [26–30]. This interaction can lead to a change in the resonant frequency of the coupled system and an increase in the effective structural damping in relation to that of the empty structure, which, in turn, can result in a relevant reduction of the structural response [31–33]. In this study, to account for HSI in the prediction of the maximum structural response, the vertical HSI-model proposed by Van Nimmen et al. [25,30] is adopted. The HSI-model allows to evaluate the variation in the modal parameters of the coupled crowd-structure system depending on the crowd density. Although the model of [25,30] still needs large-scale validation, other studies [23,24,34] support its findings. In particular, the degree to which the dynamic behaviour of the coupled crowd-structure system is modified strongly depends on the modal properties of the empty structure as well as on the pedestrian density. Considering wide ranges of modal parameters, a maximum change in the resonant frequency of about 30% is obtained, while the damping ratio is observed to increase to a value that can be 10 times higher than the one of the empty structure.

In this paper, the maximum structural response to vertical pedestrian excitation is predicted for a wide range of footbridge and crowd parameters. The structural response is evaluated combining the spectral load model of Piccardo, Tubino and Ferrarotti [13,22] with the HSI-model proposed by Van Nimmen et al. [25,30]. The spectral load model proposed in [13,22] accounts for the contribution of the fundamental mode of the footbridge and the first harmonic of the walking force. In this study, to consider a wide range of natural frequencies, the

contribution of the first three harmonics of the walking force is taken into account. One of the main limitations of the proposed procedure lies in the fact that the fundamental mode shape is assumed to be sinusoidal. Furthermore, the spectral load model can be adopted when the pedestrian load can be treated as a stationary random process, meaning that pedestrians have to be uniformly distributed on the structure and the duration of the load has to be long enough.

The paper is organized as follows. First, the procedure for the evaluation of the maximum response based on the spectral load model is discussed. Second, the coupled human-structure model is introduced and the effects of the interaction on the structural response are examined. Finally, a parametric study is performed to evaluate the structural response involving a wide range of pedestrian and footbridge parameters.

2. Evaluation of the maximum dynamic response based on the spectral model for pedestrian-induced loading

The procedure proposed by Piccardo, Tubino and Ferrarotti [13,22] for the serviceability analysis of footbridges uses the methods of linear random dynamics (e.g. [35]) to estimate the maximum structural response. The acceleration response of a system subjected to random excitation, in this case pedestrian groups modelled as a stationary random process, is statistically characterized. In the following, particular emphasis is given to the expected value of the maximum response, being the relevant one for design purposes. When a lightly-damped linear system is subjected to random excitation, the characteristics of the excitation are modified by the system, which behaves as a filter, resulting in a response similar to a narrow band random process [36,37]. According to the methods of linear random dynamics, the response of a system to random excitation, for instance in terms of accelerations, can be estimated as follows:

$$\ddot{p}_{j_{max}} = g_{\ddot{p}_j} \sigma_{\ddot{p}_j} \tag{1}$$

where j indicates the natural mode under consideration, $\ddot{p}_{j_{max}}$ and $\sigma_{\ddot{p}_j}$ are the mean value and the standard deviation of the distribution of acceleration maxima and $g_{\ddot{p}_j}$ is a peak coefficient [38–40]. The standard deviation of the output acceleration $\sigma_{\ddot{p}_j}$ is estimated from the acceleration variance $\sigma_{\ddot{p}_j}^2$, which in turn is calculated as [36,37]:

$$\sigma_{\ddot{p}_j}^2 = \int_{-\infty}^{\infty} |H_B(\omega)|^2 S_{F_j}(\omega) d\omega \tag{2}$$

where ω [rad/s] is the circular frequency, $S_{F_j}(\omega)$ is the two-sided PSD of the modal force induced by pedestrians (see Section 2.1) and $H_B(\omega)$ is the FRF of the footbridge relating the harmonic input excitation to the acceleration response. For lightly-damped systems and when the response may be assumed mainly resonant, the acceleration variance can be calculated as [35]:

$$\sigma_{\ddot{p}_j}^2 = \frac{\pi \omega_j}{2\xi_j m_j^2} S_{F_j}(\omega_j) \tag{3}$$

where ω_j, ξ_j and m_j are the natural frequency in rad/s, the modal damping ratio [-] and the modal mass [kg], respectively. The PSD of the modal force $S_{F_j}(\omega)$ and the peak coefficient $g_{\ddot{p}_j}$ are described in Sections 2.1 and 2.2.

2.1. Spectral model of the modal force

The spectral model proposed by Piccardo, Tubino and Ferrarotti [13,22] uses an analytical definition of the PSD of the modal force. A summary of the procedure for the derivation of the analytical expression of the spectral load is given here. The reader is referred to [13,22] for the detailed derivation. For a vibrating system excited by a moving load $f(x,t)$, the equation of motion can be formulated in modal coordinates. The modal force $F_j(t)$ acting on the single degree of freedom